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Simplified assessment of cable-stayed bridges buckling stability

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ABSTRACT

A simplified procedure for assessing of the overall elastic buckling stability of cable-stayed girder bridges at preliminary design stages is presented. The evaluation of the buckling modes and load factors is based on the analogy of taking a cable-stayed bridge deck as a beam–column on an elastic foundation. A new method of assessment of the model uniform continuous vertical stiffness, provided by the main span stays and reduced by the stays side span flexibility, is proposed. The results of this method are in good agreement with those of geometrical non-linear finite element analyses, typically performed at final design stages. The influence of the stay system, cable spacing, towers height, the live load pattern, and the number of the intermediate piers are finally analyzed.

1. Introduction

In the last fifty years, the range of cable-stayed bridges has been steadily increasing. Today cable-stayed bridges are typically used for spans ranging from 400 m to 1000 m. For longer spans, cable-stayed bridges compete today with suspension bridges. In fact, the recent examples of the *Stonecutters* Bridge (Hong-Kong, 2009), *Sutong* Bridge (Jiangsu, 2008), and *Russky Island* Bridge (Vladivostok, 2012) with main spans respectively of 1018 m, 1088 m, and 1104 m, proved they are feasible for spans over one thousand meters. Longer cable-stayed bridges are being designed and constructed nowadays, namely hybrid solutions, combining cable stays with suspension cables. To this respect, the *Third Bosporus* Bridge in Istanbul, with a 1408 m long main span, is a leading example.

To improve aerodynamic stability, increase the strength and reduce the dead weight, steel box girder decks have been used for these very long spans. But, for cable-stayed bridges with spans between 400 m and 600 m, and possibly to 700 m, composite steel–concrete decks are most likely the most efficient and competitive solution, as confirmed by the variety of composite steel–concrete decks built in the last twenty five years. Very slender prestressed concrete girder decks have been also used in cable-stayed bridges with spans up to 500 m.

Apart from aerodynamic stability, the key issue for long-span cable-stayed bridges is the overall safety of the bridge deck under bending and the high compressive forces induced by the staying scheme [1,2]. The full nonlinear static analysis of long-span cable-stayed bridge up to failure can be done to evaluate its overall safety. In such a case, both geometric and material nonlinearities are involved in the analysis. The geometric nonlinearities come from the cable sag effect, axial force–bending interaction effect, and large displacement effects. Material nonlinearities arise when one or more bridge elements exceed their individual elastic limits. Based on these criteria, the ultimate load-carrying capacity analysis is usually done starting from the deformed equilibrium configuration due to bridge dead loads. The results of such analysis typically showed that the overall safety of a long-span cable-stayed bridge depends primarily on the material nonlinear behavior of individual bridge elements [1–3]; geometric nonlinearities have a much smaller effect on the bridge failure behavior. They also put in evidence that an elastic stability analysis greatly overestimates the safety factor of the bridge. In fact, the results of some studies have shown large load factors against buckling failure, λ_{cr}, typically greater than six with respect to the design dead load, and well in excess of ultimate strength load factors, λ_{plast}, for the cables and main girders [2,3]. As a result, the structural stability of cable-stayed bridge decks has not received much attention from either designers or researchers.

However, there are some decisions to be taken at a very early stage of cable-stayed bridge design that should be looked carefully by designers since they increase the susceptibility to buckling of the cable-stayed deck. These are the cases of using or avoiding intermediate piers at the lateral spans, as well as choosing the cables system and defining the towers height. Therefore, during the conceptual design a quick assessment of the overall stability of the deck should be performed. For the first generation of cable-stayed bridges this was not a difficult task since they had

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widely spaced and stiffer stays along the girder that worked as supports, like intermediate piers. Hence, the deck main girders also had to be stiff and strong, as they span between stays. Buckling of the girder between stay cables was a design scenario, but the buckling load and mode shape were easily determined. And, due to the relative small spans and the stiff girders, buckling was rarely a governing design criterion for this type of cable-stayed bridges.

However, when cable-stayed bridges with continuous closely spaced cables were introduced, the calculation of the girder buckling loads became more difficult, since the buckling modes were no longer so easily defined. Yet, an approximate buckling analysis, first proposed by Tang [4], indicated that closely spaced cables provided a very effective support to the girder, and significantly increased the buckling load with respect to the earlier discrete cable supported type of structures. Some early studies also shown that decks in the 300 m span range, with closely spaced stays, had buckling load factors between 4 and 6 times the dead load, and far in excess of the ultimate strength load factors, for the cables and girders [5]. More recent studies have shown that also for very long cable-stayed spans of 1018 m the elastic buckling load factors are much higher than the material strength load factors, which govern the design [2].

During the eighties, designers made significant reductions in the depth of the girders and increased the span lengths, taking advantage of the reduced deck bending demands with closely spaced stays. A significant achievement was reached in the late eighties when very slender prestressed concrete decks started to be used. The cases of the *Diepoldsau* Bridge (Switzerland, 1985; span to depth slenderness ration of 216), *Dame Point* Bridge (EUA, 1989; deck slenderness of 264), *Skytrain* Bridge (Canada, 1990; deck slenderness of 316), *Helgeland* Bridge (Norway, 1991; deck slenderness of 313) and *Evripos* Bridge (Greece, 1992; deck slenderness of 478), are all remarkable examples of this design concept. But, even for these very slender decks, model testing and numerical calculations have shown that buckling load factors were reassuringly high [6,7].

Since then, many long-span cable-stayed bridges have been built and the several studies on the stability of cable-stayed bridges have been carried out by a number of researchers and engineers [7–19]. Most of these studies used the finite elements method (FEM) to conduct the analyses; a few used the energy method, considering both geometric and material nonlinearities in the analysis. Such an assessment is often done at the final stages of the design. Yet, the essential decisions are usually taken during the conceptual design of the structure, and should guarantee large load factors against bucking failure. To assess the deck instability at this stage a simple model of a beam–column on an elastic foundation is used, as presented in the next section.

As important for the bridge design as the safety against global deck buckling is the non-linear increase of the deck moments due to P–Δ geometrical effects. In a first approximation this increase may be evaluated by the well know expression [20,21]:

$$M^{II} = \frac{M^{I}}{1 - \frac{N_i}{N_{i,cr}}} = \frac{M^{I}}{1 - \frac{1}{\lambda_{cr}}} \tag{1}$$

Being the buckling load factor λ_{cr} defined by the maximum ratio between deck buckling load $N_{i,cr}$ and the applied normal force N_i at stay location i of the deck level, and M^I and M^{II} the 1st order and the 2nd order moments. This approximate method gives the designer an early indication of the susceptibility of his design. Non-linear increases between 10% and 20% are frequently reported for concrete cable-stayed decks, which corresponds to $\lambda_{cr} > 6$ [20].

2. Beam–column on an elastic foundation model

An energy method was first proposed by Tang [4] for the buckling analysis of bridge girders continuously supported by cables, based on the stability of a beam–column on an elastic foundation (BEF). In fact, the main span can be considered as a simple supported beam–column with vertical bending stiffness EI of the deck and elastically supported along the span by the cables. Each cable of length l_i, area A_i, modulus of elasticity E_e, and inclination α_i with the respect to the deck, provides a vertical stiffness $K_{v,i}$ given by Eq. (2), if only its elongation is considered. Since the cables are closely spaced at a centres, at the deck level, an uniform "continuous" vertical stiffness β_i can be envisaged for the elastic foundation of the beam–column (Eq. (3)). For the applied vertical load q, the deck compressive force N_i at stay's anchorage i (Eq. (4)) is given by the sum of all the horizontal compressive forces induced by the stays i to n, the longest stay on one side of the tower (Fig. 1).

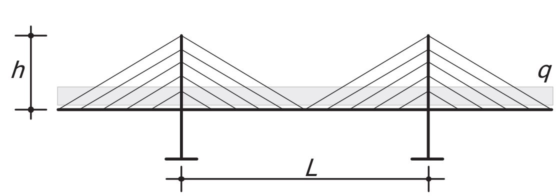
$$K_{v,i} = \frac{E_e A_i}{l_i} \sin^2 \alpha_i \tag{2}$$

$$\beta_i = \frac{K_{v,i}}{a} = \frac{E_e A_i}{l_i a} \sin^2 \alpha_i \tag{3}$$

$$N_i = \sum_{j=i}^{n} \frac{q\,a}{\tan \alpha_j} \tag{4}$$

The deck compressive force and the vertical cable stiffness increase towards the towers, and depend on the cable inclination angle α, and therefore on the stay's system. Fig. 1 presents the deck compressive forces and vertical stiffness's distributions assuming only the inclination angle of the cables is changed between three typical stay systems. To this respect the harp system is the less efficient. It provides the weakest vertical elastic support and introduces the highest compressive forces on the deck N_{max}, at towers intersection (Eq. (5)). The fan system is the most efficient, with half of N_{max} for the same tower's height (Eq. (7)).

The semi-fan system has a comparable efficiency to the fan system when the stays are anchored in the upper half of the towers, introducing a maximal compressive force at the towers intersection given by Eq. (6). Thus, this system leads to a maximum deck compressive force only 23% higher than the fan system and provides a little higher vertical stiffness at the central part of the span, essential for preventing deck buckling. Moreover, in practice, the fan system has the important drawback of having to anchor or deflect all stays at the top of the towers. These are the key reasons for adopting the semi-fan stay's system in nearly all cable-stayed bridges exceeding 400 m long main span.



Harp system $$N_{\max} = \frac{qL^2}{4h} \tag{5}$$