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Constitutive model selection for unreinforced masonry cross sections based on best-fit analytical moment–curvature diagrams

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ABSTRACT

A significant number of macroscopic constitutive models is available to simulate the behaviour of unreinforced masonry (URM) in compression, but they produce different predictions of sectional response to eccentric compressive loading. In this paper, nonlinear moment–curvature analysis of rectangular URM cross sections is presented to explore the influence of alternative constitutive models and ultimate limit state assumptions in terms of post-peak strength degradation and strain ductility. Theoretical moment–curvature diagrams are quantitatively compared to experimental data at multiple levels of load eccentricity through goodness-of-fit measures. Different constitutive models are found to produce a good simulation of experimental moment–curvature behaviour, depending on whether experimental data are disaggregated in terms of masonry type or not. Moment–curvature diagrams are derived by keeping constant either the magnitude or the eccentricity of axial load. Finally, the effects of tensile properties and model error of theoretical moment–curvature diagrams are assessed.

1. Introduction

Unreinforced masonry (URM) constructions are a large fraction of the worldwide built heritage and most of them need to be assessed and protected against a number of hazards. This calls for the use of demand-capacity models that are able to provide realistic predictions of structural response with acceptable computational work.

When evaluating the structural safety of an URM construction, several components such as arches, vaults, pillars and walls can be subjected to some loading conditions that produce eccentric compression. In the case of buildings, a masonry wall may be subjected to eccentric compression as a result of wind loads, imperfections, construction deviations, eccentricities of upper-storey walls due to reduced thickness, second order effects, gravity loads transmitted over a fraction of wall thickness that supports the floor system, lateral pressures induced by thrusting elements, backfill soil or other loads, in-plane seismic actions, and out-of-plane seismic actions, transmitted by lower structures and/or flexible floors. The latter loads are, for example, a typical source of local collapse mechanisms in historic buildings without proper seismic detailing (see e.g. [1,2]). If a masonry wall subjected to eccentric compression in its own longitudinal plane is considered, its overall mechanical response may be analysed through a one-dimensional element provided that the wall has a relatively high slenderness (i.e. a large height-to-length ratio) and is made of a regular assemblage of masonry units and mortar joints. Hence, the behaviour of that beam-type element may be derived from that related to its cross section by assuming a macroscopic constitutive model for masonry in compression and plane cross sections after flexural deformation [3–6]. The same assumption may be used, for instance, to assess a masonry wall subjected to eccentric gravity loads (i.e. compressive loading with eccentricity out of longitudinal plane) if a low level of constraint is provided by transverse walls or the ratio of the distance between orthogonal walls to the height of the wall is rather high. Nonetheless, in that loading condition, a large vertical slenderness in the transverse plane of the wall (i.e. a large height-to-thickness ratio) induces a premature collapse of the wall because buckling occurs before the full plastic capacity of cross section is exploited. In line of principle, the use of a beam-type model may become questionable if a squat wall or an irregular masonry assemblage such as multi-leaf stone masonry is considered. In those conditions, complex modelling strategies may be required (see e.g. [7]).

This study deals with situations where one-dimensional elements can be employed to get realistic response predictions under eccentric loading. This allows the use of simplified capacity models, see e.g. [8,9], where masonry is ideally modelled as an

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equivalent continuum medium subjected to average stresses and strains. To that aim, several macroscopic constitutive models are available in the literature and may be used to develop 'mechanics-based' macro-element capacity models. In that kind of models, the mechanical behaviour of masonry is explicitly taken into account and processed through direct integration of sectional response over a finite volume of macro-element. On the contrary, current building codes [10–13] do not specifically account for constitutive modelling of masonry and provide some modelling rules and verification criteria for piers and, sometimes, spandrels.

In many cases, the tensile strength of masonry is significantly lower than compressive strength, allowing the use of no-tensile-resistant (NTR) models. Sectional response to eccentric compression is defined through a capacity model composed of two parts: axial load–bending moment interaction domains at multiple limit states; moment–curvature diagrams at multiple levels of axial load or load eccentricity. Such a capacity model clearly depends on the uniaxial stress–strain relationship assigned to masonry (especially in compression), which needs to be appropriately selected. In recent years, Parisi and Augenti [14] assessed the influence of several macroscopic constitutive models on the flexural strength of rectangular URM cross sections, evidencing some potential effects of compressive strength degradation especially at low-to-medium levels of axial load. From the viewpoint of flexural deformation capacity, some interesting studies were carried out to investigate the nonlinear moment–curvature response, either theoretically or experimentally (see e.g. [3,4,6,15,16]). Nevertheless, a comprehensive analysis is still required to assess the ability of a significant suite of constitutive models to reproduce the experimental moment–curvature response. This study attempts to answer this question by performing a quantitative theoretical–experimental comparison based on a number of goodness-of-fit measures. Such measures allow the identification of the constitutive models that provide the best simulation of experimental moment–curvature behaviour. In this context, the implications of alternative assumptions for ultimate strain of masonry are also investigated. Finally, the model error of analytical moment–curvature diagrams corresponding to the best-fit constitutive models is characterised and the influence of tensile properties is evaluated.

2. Methodology

The first part of this research was mainly based on the following steps: (1) selection of nonlinear macroscopic constitutive models with different features in terms of elastic behaviour, plastic behaviour and deformation capacity; (2) nonlinear moment–curvature analysis of rectangular URM cross sections based on numerical integration of balance equations; (3) quantitative comparison of theoretical moment–curvature diagrams to experimental data on clay brick masonry and calcarenite stone masonry.

Two incremental iterative procedures for nonlinear moment–curvature analysis were developed in MATLAB® [17], as presented in Appendices A and B. The first procedure was used to derive moment–curvature diagrams under a given level of load eccentricity. That assumption was consistent with the type of experimental data provided by past eccentric compression tests, where compressive load was increased by keeping constant its location over the cross section. Specifically, experimental data were related to three levels of load eccentricity. The second moment–curvature analysis procedure was developed to obtain diagrams under a given level of axial load, which is a typical assumption in structural analysis of buildings and is consistent with the use of axial load–bending moment interaction domains.

Moment–curvature analysis was first carried out by considering the full constitutive models selected from the literature, most of which have different levels of strain ductility. In order to compare numerical results in similar conditions, moment–curvature analysis was repeated for two variants of stress–strain relationships: constitutive models truncated to the same level of post-peak strength degradation on the softening branch (strength-based truncated variant); constitutive models truncated to the same level of strain ductility (ductility-based truncated variant). Post-peak strength degradation of strength-based truncated models was assumed to be 20% in accordance with capacity modelling of masonry walls and buildings at the ultimate limit state (ULS), see e.g. [9,13]. Such an assumption is also used to define the ultimate strain of quasi-brittle materials such as masonry, as reported for instance by Brencich and de Felice [15]. Conversely, the ductility-based truncated variants of constitutive models were characterised by a ductility equal to 2 which is approximately the minimum ductility level of the complete constitutive models. It will be shown that the implementation of constitutive models with different assumptions in terms of ultimate strain may lead to strongly different predictions of the post-peak moment–curvature behaviour.

Theoretical results were compared to experimental data collected from the literature. Those data were grouped according to the following alternative rules: collection of all experimental data associated with different types of masonry; disaggregation of the database in two subsets, each of them being related to a single masonry type. Based on three goodness-of-fit measures, the constitutive models associated with theoretical moment–curvature diagrams that better fit experimental data were identified. This allows one to select the macroscopic constitutive models on the basis of their effects on the simulation of moment–curvature behaviour.

Finally, the model error of analytical moment–curvature diagrams corresponding to the best-fit constitutive models was statistically characterised in terms of mean and coefficient of variation (CoV). The paper ends with a discussion on moment–curvature diagrams corresponding to fixed levels of axial load and those associated with a non-zero tensile strength of masonry. The effects of tensile mechanical properties on moment–curvature diagrams were evaluated by assuming a linear elastic behaviour up to the peak tensile strength and a linear softening behaviour up to the ultimate tensile strain.

3. Stress–strain diagrams for unreinforced masonry in compression

Numerical integration of 26 constitutive models selected from the literature was performed to derive moment–curvature diagrams by keeping constant the eccentricity (Appendix A) or magnitude (Appendix B) of axial load. The constitutive models considered in the analysis are the following:

- the elastic-perfectly plastic (EPP) model with limited ductility, which is allowed by several building codes and often adopted in engineering practice for design/assessment purposes [13,14];
- the nonlinear elastic-perfectly plastic (NEPP) model with limited ductility, which is denoted as parabolic rectangular stress–strain relationship in Eurocode 6 (EC6) [12];
- the Kent–Park model [18] in its two variants proposed by Priestley and Elder [19] for concrete block masonry (herein abbreviated as 'Kent and Park_PE') and Brencich and de Felice [15] for clay brick masonry (herein abbreviated as 'Kent and Park_BD');
- the Arya–Hegemier model [20] related to concrete block masonry;
- the modified Sargin model [21] proposed by Cavaleri et al. [4] for calcarenite stone masonry;
- the Capozucca model [22] related to clay block masonry;