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Membrane stress analysis of collapsible tanks and bioreactors

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ABSTRACT

Collapsible tanks, vessels or bioreactors are finding increasing usage in small/medium scale processes because they offer flexibility and lower cost. However, if they are to be used at large scale, they need to be shown capable of handling the physical stress exerted on them. Because of their nonconventional shape and non-uniform pressure distribution, thin shell analysis cannot be used in calculating their stress. Defining curvature in terms of pressure addressed these challenges. Using curvature and numerical analysis, the membrane stress in collapsible tanks designed as bioreactors of volumes between 100 to 1000 m^3 were calculated. When the liquid/gas height and static pressure are known, an equation for calculating tension per length was developed. An equation that could calculate the liquid height from the bioreactor's volume, dimensions and working capacity was generated. The equation gave values of liquid height with a maximum deviation of 3% from that calculated by curvature analysis. The stress values from the liquid height and tension equations had a maximum deviation of 6% from those calculated by curvature analysis. The calculated tensile stress in a 1000 m^3 collapsible tank was 14.2 MPa. From these calculations, materials that optimize both cost and safety can be selected when designing collapsible tanks.

1. Introduction

Collapsible tanks or vessels are vessels made from materials that are flexible, easily deformable, and light. Without a rigid supporting system collapsible tanks normally take the shape of a pillow when filled with fluid, so they are also called pillow or bladder tanks. Collapsible tanks are used as water storage vessels, fuel storage vessels, transportation vessels, sewage tanks, food storage vessels, chemical storage vessels, bioreactors for e.g. ethanol [1] or biogas production [2] etc. The benefits of using collapsible tanks over rigid tanks include process flexibility, ease of transportation, installation, portability and low cost. Collapsible tanks come in sizes suitable for use at small, medium [3] and large scale. Collapsible tanks are usually designed with their end use in view. One common question when using these collapsible tanks especially at larger scale is if the materials used for constructing them can tolerate the stress caused by the high pressure and the large liquid volume. Despite their wide application, there is no scientific publication on how to calculate the stress in these containers to safeguard against failure.

When collapsible tanks are used for storing or processing fluids, they can be designed as pressure vessels. A pressure vessel is a vessel that can withstand the internal pressure that is acting on it [4]. According to its wall thickness, pressure vessels can be classified as thick or thin walled. A pressure vessel is thin walled if the ratio of its wall thickness to its radius is less than or equal to 1/20 [4] . Thin walled vessels offer no resistance to bending, so the stress on it is distributed through its thickness, resulting in only membrane stresses, while thick walled vessels offer resistance to bending, so they have both membrane stresses and bending stresses [4]. Thus, collapsible tanks are thin-walled pressure vessels. Thin shell theory is normally used for calculating the stress in thin walled pressure vessels under constant internal pressure [4]. Using the thin shell theory in calculating the membrane stress in collapsible tanks would not give accurate values. Some reasons for this are; collapsible tanks will not have the same pressure at all points, and they will not have a specific shape at all times because their shape will change with changes in pressure and the volume of fluid in them [5].

Accurate determination of the stresses in any pressure vessel is essential to safeguard against failure [6] which could occur when the membrane stress in the vessel has exceeded the material's intrinsic tolerance values as determined by its Young's modulus [7]. One challenge why the thin shell theory cannot be used to calculate the stress that would act on collapsible tank is that their

Abbreviations: DEs, differential equations; WC, working capacity.

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Nomenclature

k	Curvature
s	Arc length
P_0	Static pressure
h	Y_- liquid height
Hg	Gas height
α	Directional angle of the tangent to the curve
T	Membrane stress force or tension per unit length
W	Width of collapsible tank
L	Length of collapsible tank
p	Perimeter of collapsible tank
A_L and A	Liquid and total cross sectional area of collapsible tank

shape would change with variations in the pressure and volume of the fluid in these tanks. Defining their curvature as a function of the fluid property of interest, which in this case is pressure, helps to overcome this challenge. This also helps in handling the second challenge of non-uniform pressure distribution in collapsible tanks. In this work, the membrane stress associated with collapsible tanks and their geometry was determined using their curvature and numerical analysis. The results gotten from the analysis performed on collapsible tanks can assist in determining their strength, how suitable they would be for large scale purposes, how they can be designed to minimize the stress in them, and what materials are suitable for making them.

2. Methods

The geometry of a collapsible tank as determined by curvature is shown in Fig. 1. The stress analysis performed in the direction of the width of a collapsible tank is termed circumferential, while that one performed in the direction of the length is termed longitudinal. The key assumptions used for the analysis performed in this work are; (a) the weight of the material used for constructing the collapsible tank can be neglected, (b) the material is infinitely flexible in the direction of curvature, (c) the collapsible tank membrane stress can be analysed using one plane at a time, and (d) the sum of the static pressures generated by the two directions of curvature gives the overall static pressure acting on the collapsible tank. As failure due to tensile stress has a higher chance of occurring in areas with less material reinforcement [7], curvature analysis performed in this paper best describes regions far from the edges of the collapsible tanks. In these areas, it can be assumed that there would be no interaction of the two orthogonal axes and the joints in the collapsible tank to increase its strength.

2.1. The shape of a collapsible tank as defined by its curvature

To find the shape of a collapsible tank, a reference frame was chosen in which position and altitude were considered using the top height of the liquid level as the origin [8]. Every height below the top of the liquid level was negative, while only the gas height (if present) was positive (Fig. 2).

The curvature (k) of any shape or line was defined using Eq. (1), where α is the directional angle of the tangent to the curve and s is the arc length [9]. However, as the pressure is the source of the curvature, the curvature was expressed in terms of the pressure according to Eq. (2), where P_0 is the static pressure above the liquid (N/m^2), y_- is the liquid height (m) which is defined with respect to the y axis according to Eq. (3), g is gravity (m/s^2), ρ (rho) is density (kg/m^3) and T is the membrane stress force or tension per unit length (N/m). The differential equations (DEs) relating the curvature, the arc length and the x and y coordinate functions for the curve are shown in Eq. (4) [10].

$$k = \frac{d\alpha}{ds} \tag{1}$$

$$k = \frac{P_0 - y_- g\rho}{T} \tag{2}$$

$$y_- = \begin{cases} y, & y \le 0 \\ 0, & y > 0 \end{cases} \tag{3}$$

$$\begin{cases} \dfrac{d^2x}{ds^2} = k\dfrac{dy}{ds} \\ \dfrac{d^2y}{ds^2} = -k\dfrac{dx}{ds} \end{cases} \tag{4}$$

2.2. Numerical analysis

A system of 4 first order DEs ($U_1(s)$:$U_4(s)$), were defined to reduce the 2 second order DEs in Eq. (4) to their first order forms [11]. The defined functions are shown between Eq. (5)–(8).

$$U_1(s) = x(s) \tag{5}$$

$$U_2(s) = y(s) \tag{6}$$

$$U_3(s) = \frac{dU_1}{ds} = \frac{dx}{ds} \tag{7}$$

$$U_4(s) = \frac{dU_2}{ds} = \frac{dy}{ds} \tag{8}$$

The DEs generated by substituting the defined functions (Eq. (5):(8)) and Eq. (2) into Eq. (4) results in a system of first order DEs as shown in Eq. (9). The resulting system of DEs (see Eq. (10)) was solved numerically using MATLAB® ODE45 solver. The calculations were performed using initial values of $U_1(0) = 0$, $U_2(0) = Hg$, $U_3(0) = 1$, and $U_4(0) = 0$.

$$\begin{cases} \dfrac{dU_3}{ds} = \dfrac{d^2x}{ds^2} = k\dfrac{dy}{ds} = kU_4(s) = \dfrac{P_0 - U_2 g\rho}{T}U_4(s) \\ \dfrac{dU_4}{ds} = \dfrac{d^2y}{ds^2} = -k\dfrac{dx}{ds} = -kU_3(s) = -\dfrac{P_0 - U_2 g\rho}{T}U_3(s) \end{cases} \tag{9}$$

$$\begin{cases} \dfrac{dU_1}{ds} = U_3(s) \\ \dfrac{dU_2}{ds} = U_4(s) \\ \dfrac{dU_3}{ds} = \dfrac{P_0 - U_2 g\rho}{T}U_4(s) \\ \dfrac{dU_4}{ds} = -\dfrac{P_0 - U_2 g\rho}{T}U_3(s) \end{cases} \tag{10}$$

2.3. Stress and shape determination

The numerical results from equation 10 was used to analyse the relationship between the membrane stress (T), the static pressure (P_0), the gas height (Hg) and the shape of the collapsible tank. The stress (T), static pressure (P_0) and gas height (Hg) (see Eq. (10)) were used as control parameters to generate the corresponding collapsible tank shape (x,y). From this shape, other properties of the collapsible tank such as width, liquid depth, and cross sectional area were calculated. The width (W) of the collapsible tank was determined as shown in Eq. (11), when carrying out the circumferential stress analysis.

$$W = 2x(s) \text{ at the point where } \frac{dx}{ds} = 0(s_1 \text{in Fig. 2}) \tag{11}$$