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The characteristic flow equation: A tool for engineers and scientists

Rich Lacey*

TRI Environmental, 9063 Bee Caves Road, Austin, TX 78733, USA

A R T I C L E I N F O

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A B S T R A C T

A relationship between pressure and flow rate is presented for describing the flow regime for laboratory permeability tests. The equation is unique to the test specimen, the fluid and the apparatus used, and is termed the Characteristic Flow Equation or “CFE”. A CFE can be established for any laboratory conductivity test including rigid-wall permeability, permittivity and transmissivity tests.

The Characteristic Flow Equation is a quadratic formula consisting of two terms; one where head loss is directly proportional to the flow velocity and one where head loss is proportional to the velocity squared. These terms account for two sources of head loss that accumulate as the fluid passes into, through, and out of, the material; one that is due to viscous resistance referred to as the “friction term”, and one that is due to losses of kinetic energy referred to as the “inertia term”.

The definitions of the variables that comprise the two coefficients in a CFE are a function of the hydraulic details of the test method and the associated measurement parameters.

This paper presents a review of research on non-linear flow in porous media, the application of the Characteristic Flow Equation (CFE) for determining the hydraulic permittivity and transmissivity of geosynthetic materials using air as the test fluid instead of water, and an algorithm for determining the Percent Open Area of woven geotextiles.

An example showing how the CFE theory can be used to evaluate the hydraulic behavior of a laboratory conductivity test, as well as other applications of the CFE and the potential benefits of testing with air instead of water are discussed in closing.

1. Introduction

Fig. 1 presents examples of typical flow rate versus head loss results for laboratory permeability tests. Several international test method standards refer to the initial, straight-line portions of these curves as the laminar region. These include: ASTM D 4491 (ASTM D 4491, 2015) Permittivity, ASTM D 4716 (ASTM D 4716, 2014) Transmissivity, ASTM D 6574 (ASTM D 6574, 2013) Radial Transmissivity, ISO 11058 (ISO 11058, 1999) Permittivity, ISO 12958 (ISO 12958, 1999) Transmissivity and AS3706.9 (AS3706.9, 2001) Permittivity to name a few. The concave downward portion is referred to as the “non-laminar” region in both ISO standards.

The emphasis in all of these standards, is to obtain the permeability in the “laminar” region unless the index test is performed at a specified head loss, such as 50-mm. This paper supports this Darcy theory of the linear head loss versus flow rate relationship, and amends it with the additional head loss caused by the loss of the kinetic energy of the pore fluid to the media structure.

2. Non-linear flow in porous media literature review

The general consensus among most engineers and scientists is that nonlinear flow is turbulent. However, there is an entire field of research focused specifically on nonlinear flow behavior where alternate explanations have been formulated. The actual source of the nonlinearity remains to be identified convincingly, but two of the most common theories are that the nonlinear response is due to:

- Convective accelerations and decelerations of the fluid flow due to converging and diverging ducts as well as changes in flow direction in the tortuous medium geometry, i.e., changes in inertia.
- Dynamic form drag involving separation of boundary layers and the formation of low pressure wakes on the downstream side of solid objects.

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* Tel.: +1 512 263 2101; fax: +1 512 263 2558.
E-mail address: rlacey@tri-env.com.

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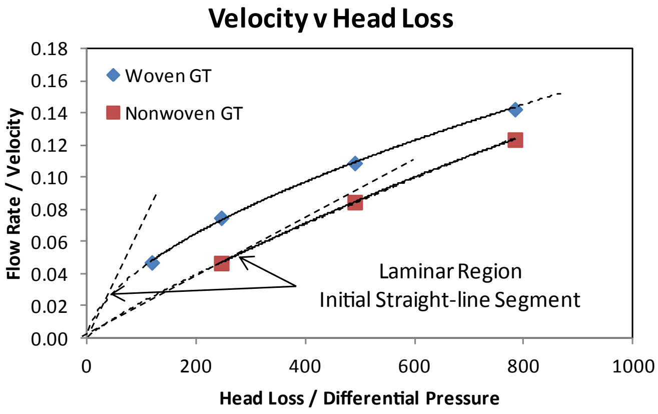

Fig. 1. A typical flow rate versus head loss permeability test result.

The following literature review is focused on the researchers' study of nonlinear flow and turbulence. Several of the proposed solutions for the nonlinear term are also presented.

The nonlinear head loss versus flow rate relationship has been observed in measured laboratory permeability data by various researchers since the early 1900's. One of the first to formulate a quadratic equation to represent non-linear flow in a porous media was Phillip Forchheimer (1901). The Forchheimer Equation is frequently presented as:

$$\frac{\Delta P_{HL}}{\Delta L} = \frac{\boldsymbol{\mu}}{K_d}\cdot V1 + \beta\cdot\boldsymbol{\rho}\cdot V1^2 \tag{1}$$

The "Beta Factor", β, is commonly referred to as the "Forchheimer Coefficient", the "Non-Darcy Coefficient" or the "Inertia Factor", and is an ongoing topic of research of non-linear flows through packed beds. The applications include the petroleum industry for estimating the intake capacities of oil wells, and the chemical processing industry for designing scrubbers and catalytic reactors. The associated laboratory testing is typically performed with cylindrical, rigid-wall permeameters with diameters of 50–200 mm, ranging in length from 1 to 6 m. These cylinders are usually packed with sands, gravels, glass beads, ceramic beads, marbles, and other types of uniformly graded particles of varying diameters and angularity with porosity ranging between 0.3 and 0.5. The pressure head measurements are of the head losses across the test devices, while some have measurements within the flow section via internal manometer taps.

This field of study has adopted a standard format of the nonlinear data referred to as the "Forchheimer Plot". This plotting technique is demonstrated with the geonet transmissivity test data reported later in this paper.

- **Nonlinear Flow and Turbulence** – Huang and Ayoub (2008) conclude that "Derivation of the Forchheimer equation from the Navier-Stokes equation reveals that the nature of the Forchheimer flow regime is laminar with inertia effects. The inertia resistance factor β can be used to characterize this flow regime and is therefore an intrinsic property of the porous media." and "Despite the diverse opinions on the origin of the nonlinearity, it is now generally agreed that the quadratic term in the Forchheimer equation is associated with the inertia effect in the laminar regime and is fundamentally different from the quadratic velocity dependence for turbulent flow."

In addition, Balhoff and Wheeler (2009) state that nonlinearities associated with the Forchheimer equation occur at velocities well-before, and unrelated to, turbulence.

- **Geotextile Permeability and Darcy's Law** – van der Sluys and Dierickx (1987) address the nonlinear behavior of geotextile permittivity tests, but not the associated quadratic equations. They conclude that "The theoretical models describing laminar flow show little correspondence to our experimental data. Even at low flow velocities no laminar flow conditions were observed." In other words, even at low velocities, the flow response was nonlinear.
- **Geotextile Permeability and Temperature Corrections** – Two papers from 1994, Dierickx and Leyman (1994), and Bezuijen et al. (1994), on the temperature correction for water permeability of geotextiles, refer to a quadratic equation where the linear term is identified as the "laminar" term, and the nonlinear term as the "turbulence" term. The viscosity variable is shown to be present in the laminar term, where both papers conclude that the temperature correction should be applied. The nonlinear term is referred to as "independent of viscosity", but the fluid density is not mentioned.

ISO 11058 for permittivity includes a note to this effect in Section 5.2.2: "As the temperature correction (see annex A) relates only to laminar flow, it is advisable to work at temperatures as close as possible to 20 °C to minimize inaccuracies associated with inappropriate correction factors, should the flow be nonlaminar."

- **Geotextile Falling Head Tests** – Bezuijen's paper (Bezuijen, 1998) on turbulence and dynamics in the falling head test includes and references the Forchheimer equation, with experimentally obtained coefficients. The ISO standard for permittivity, ISO 11058, presents a quadratic equation for calculating the velocity index from falling head data. However, neither of these identify where the viscosity and/or density parameters occur in the equations.
- **Exponential Nonlinear Flow Analyses** Giroud and Kavazanjian (2014) employed the inverse approach to analyze nonlinear flow behavior, with the flow velocity as a function of gradient similar to Fig. 1. The form of this equation is:

$$v = \lambda\cdot i^m$$

The coefficients, λ and m, for the inverse, exponential form of the quadratic CFEs presented in this paper are compared with the values generated by Giroud and Kavazanjian (2014) in Table 1. The two components of the head loss cannot be differentiated with this analysis.

2.1. Solutions for beta

The proposed solutions for the Forchheimer Beta coefficient are numerous and widely-varied. Some are simply related to the particle diameter, while others are more complex, and are related to the porosity, particle diameter, and permeability. Some of the solutions are empirically derived from experimental data, while others are based on complex mathematical models of the porous media.

- Geertsma (1974) proposed an empirically derived equation for Beta based on the porosity, n, and formation permeability from experiments performed on consolidated sandstone, K_d:

$$\beta = \frac{0.005}{n^{5.5}\cdot K_d^{0.5}}\ when\ \ i = f(V1) \tag{2}$$