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Stochastic optimization of mine production scheduling with uncertain ore/metal/waste supply

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ABSTRACT

Optimization of long-term mine production scheduling in open pit mines deals with the management of cash flows, typically in the order of hundreds of millions of dollars. Conventional mine scheduling utilizes optimization methods that are not capable of accounting for inherent technical uncertainties such as uncertainty in the expected ore/metal supply from the underground, acknowledged to be the most critical factor. To integrate ore/metal uncertainty into the optimization of mine production scheduling a stochastic integer programming (SIP) formulation is tested at a copper deposit. The stochastic solution maximizes the economic value of a project and minimizes deviations from production targets in the presence of ore/metal uncertainty. Unlike the conventional approach, the SIP model accounts and manages risk in ore supply, leading to a mine production schedule with a 29% higher net present value than the schedule obtained from the conventional, industry-standard optimization approach, thus contributing to improving the management and sustainable utilization of mineral resources.

1. Introduction

Open pit mines are the largest source of metal and energy resources. Optimizing their long-term production scheduling is a complex and intricate process that defines the sequence of ore and waste extraction during the life-of-mine and up to an ultimate pit limit. This optimization process deals with the management of cash flows in the order of hundreds of millions of dollars and is heavily impacted by uncertainty in the metal/ore and waste forecasted to be produced from a pit in both valuations and operation. To deal with this uncertainty, a two stage stochastic integer programming formulation is tested in an application at a copper mine. Mine design and production scheduling is traditionally divided, for practical reasons, into two major tasks: first, an ultimate economic boundary beyond which mining becomes uneconomical is delineated and then, the extraction sequence of the set of selective mining units (SMU) contained inside this final boundary or pit is defined. Both problems are typically formulated such that an optimum maximum economical return for the mine is obtained. The optimum open pit mine production schedule is defined as the sequence of extraction that maximizes the present value of the project. This task is one of the most challenging and important in the mine planning framework as it defines the ore supply produced over the life-of-mine (LOM) and consequentially has a substantial impact on the net present value (NPV) of the project. The conventional mine design and production framework defines the extraction sequence considering a single, average type of orebody model as input. The weaknesses of such an approach are well documented; for example, it has been demonstrated that the use of an average model of the orebody as an input for mine planning optimization algorithms as practiced conventionally, may lead to misleading forecasts and assessments [1–3]. This finding makes clear that there is a need to address mine production scheduling with stochastic optimization formulations, which by construction use as input a set of equally probable representations (scenarios) of the orebody being considered that reproduce its actual spatial variability, and this set of scenarios is able to directly incorporate the related uncertainty into the production scheduling formulation.

Different stochastic approaches have been considered in order to provide more robust solutions dealing with uncertainty. Early efforts utilizing mixed integer programming (MIP) with a probabilistic approach for multi-element deposits are shown, where probabilities were computed from simulated models; the final schedule uses these probabilities to maximize the NPV of the project [4]. A major next step in fully utilizing the information available in a set of simulated scenarios of a mineral deposit to be scheduled is presented in the development of a fully stochastic scheduling optimization approach using a simulated annealing algorithm to obtain a stochastic mine production schedule [5,6]. The proposed

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approach is divided into three stages; in the first stage, optimum mining rates are defined using an LP formulation, in the second stage the rates are utilized to schedule a set of simulated ore bodies, and the schedules are then used in a final stage in which a stochastic schedule is obtained by using a simulated annealing algorithm and then scheduled as an input. The study shows a 28% difference in NPV as compared to the conventional schedule. Variants of this approach have been used to further assess its performance in a copper deposit and show an improvement of 25% in NPV when compared to the conventionally derived schedule [7,8].

Stochastic integer programming or SIP has been used as a framework to optimize long-term mine production schedules that maximize NPV while considering several possible simulated scenarios of the orebody and simultaneously optimize cut-off grades or maximize NPV while minimize deviation from production targets using different penalties defined for deviations of different targets [9–12]. Another relevant study combines the use of the SIP formulation with the use of simulated future grade control data, updating simulated models to then produce an optimum stochastic mine schedule [13]. These studies show an increase in NPV in the same order as the studies using simulated annealing that were mentioned above.

Despite substantial monetary benefits, the application of stochastic schedulers is relatively recent and the value of this solution when applied to different types of deposits is still not completely understood. It is therefore important to test the application of stochastic schedulers in different types of mineralizations to assess the complexities and intrinsic characteristics of such schedulers. In the present study, the approach presented by Ramazan and Dimitrakopoulos is applied to a low-grade variability copper deposit [11]. The study tests the approach, quantifies the associated value of the stochastic solution, assesses the risk profile of pertinent mining parameters and, finally, analyses the results to propose future improvements. The study aims to explore the method's capability to incorporate geological uncertainty in the mine production scheduling problem formulation and to manage the risk of deviating from production targets. The following sections describe the stochastic integer programming formulation in detail, present its application in a copper deposit and the results obtained, and compare the results with those obtained by a conventional scheduler. Conclusions and discussion follow.

2. SIP formulation for long-term open pit production scheduling

Stochastic mathematical programming provides new avenues to address mine planning optimization through the direct incorporation of uncertainty of ore supply in the formulation of the problem, and minimize the risk of not meeting the mine production targets, while maximizing total discounted cash flows [9]. More specifically, in the mine production scheduling case, the decision required is which time period each block of the model of the orebody considered should be mined, important so as to maximize the overall discounted value of the project (NPV), subject to slope, reserves, and processing and mining capacity constraints. Conventionally, the set of blocks available to be scheduled are the ones contained within the ultimate pit. The SIP formulation presented herein includes uncertainty in the formulation of the problem by jointly considering a set of different and equally probable stochastic simulated orebody realizations (scenarios) in the optimization process.

2.1. Economic value of a block

The optimization process considers the economic value of the set of blocks to be scheduled. The expected value of a block $E\{V_i\}$ is defined herein using its expected return NR_i, which is defined as the expected gain from a set of possible stochastic simulated grades (metal content) for the given block i. The value of a given block j is therefore defined as

$$E\{V_i\} = \begin{cases} NR_i - MC_i - PC_i, & \text{if } NR_i > PC_i \\ -MC_i, & \text{otherwise} \end{cases} \tag{1}$$

given that

$$NR_i = T_i \times G_i \times REC \times (\text{Price} - \text{Selling Cost}) \tag{2}$$

where NR_i represents the expected net revenue; MC_i the mining cost; PC_i the processing cost; T_i the tonnage; G_i the grade and REC the recovery.

2.2. Objective function

The formulation aims to maximize the NPV of the mine by minimizing the risk of falling short of previously defined production targets. It includes two possible destinations for a block, processing plant or waste dump. The objective function includes two components and it is

$$\text{Max} \sum_{t=1}^{p} \left[\underbrace{\sum\nolimits_{i=1}^{N} E\left\{(\text{NPV})_i^t\right\}}_{\text{Part A}} * b_i^t - \underbrace{\sum\nolimits_{s=1}^{m} \left(c_u^{to} d_{su}^{to} + c_l^{to} d_{sl}^{to} \right)}_{\text{Part B}} \right] \tag{3}$$

where i is the block identifier; t is the time period; to flags the ore production target type; l stands for lower bound; u stands for upper bound; s stands for the simulation number; p is the maximum number of scheduling periods; N is the total number of blocks to be scheduled; b_i^t is a variable representing the portion of block i to be mined in period t; if defined as a binary variable it is equal to 1 if the block i is to be mined in period t and equal to 0 otherwise; $E\{(\text{NPV})_i^t\}$ is the expected NPV to be generated by mining block i in period t; it is computed as the discounted value of Eq. (1); c_u^{to} is the unit cost for excess of ore production; d_{su}^{to} is the excess amount of ore production in period t considering simulation s; c_l^{to} is the unit cost for the deficient ore production; d_{sl}^{to} is the deficient amount of ore production in period t considering simulation s.

The first component (Part A) in Eq. (3) contributes to the maximization of NPV of the project. The expected NPV of a block is computed as the expected present value if the block is mined in period t, considering all simulated values. The second part (Part B) is responsible for minimizing deviations from ore production targets, also managing the distribution of risk within and between periods over the LOM. Risk management is accomplished through the use of a geological discount rate (GDR), which discounts over time the penalties applied to the unit cost deviations as explained below. The initial penalties for excess, c_u^{0o}, or shortage production, c_l^{0o}, are user defined and should be at the same order of magnitude as the first part of the objective function to ensure the second part is being properly considered. The impact of discounted penalties is a progressive decrease in the unit cost over the periods. This setup ensures that not only mining blocks with high grades and economic value will be mined as early as possible (earlier production periods), but also less "risky" mining blocks will be scheduled in the same early periods, therefore decreasing the risk of not attaining the planned targets and guarantying the minimization of production target deviations. This means that the resulting group of mining blocks to be mined in a given year will be different from those in conventional optimization formulations, in the sense that the grouping of blocks is not a function of some average discounted economic value, but a "blend" of high economically valued blocks with blocks of high probability to have high economic values.