International Journal of Solids and Structures 96 (2016) 240–253

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Exact solution based finite element formulation of cracked beams for crack detection

Ugurcan Eroglu, Ekrem Tufekci*

Istanbul Technical University, Faculty of Mechanical Engineering, Gumussuyu 34437, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 2 December 2015
Revised 3 May 2016
Available online 2 June 2016

Keywords :
Cracked beam
Crack detection
Genetic algorithms
Beam theory
Finite element method
Experimental modal analysis

ABSTRACT

In this study, a new finite element formulation is presented for straight beams with an edge crack, including the effects of shear deformation, and rotatory inertia. The main purpose of the study is to present a more accurate formulation to improve the beam models used in crack detection problems. Stiffness matrix, consistent load vector, and mass matrix of a beam element is obtained using the exact solution of the governing equations. The formulation for frame structures is also presented. Crack is modelled utilizing from the concepts of linear elastic fracture mechanics. Several numerical examples existing in the literature related to the vibrations of such structures are solved to validate the proposed model. Additionally, an experimental modal analysis is performed to see the superiority of the present method for high modes of vibration, which are generally not taken into account in crack detection problems. The inverse problem is also solved using a well–known optimization technique called genetic algorithms. Effects of shear deformation, rotatory inertia, and number of natural frequencies considered, on the accuracy of the estimation of crack parameters are investigated. It is found that considering more number of frequencies yields better estimation of crack parameters, but require a better modelling of the dynamics of the beam. Therefore, the present formulation is found to be an essential tool in crack detection problems.

1. Introduction

Beams are one of the simplest structural elements. The simplicity of the governing equations makes the static, and dynamic behavior of beams easy to analyze and manipulate. Also, the manufacturing process is easy due to their simple geometries. Thus, beams have been used in many structures from different engineering disciplines such as civil, mechanical, ocean, aerospace to carry and/or transfer loads over the years. Since the application area of beams and beam-like structures is extensive, it is obvious that monitoring their health is a very important issue to maintain the functions of the entire structure, and to ensure the safety. Despite the advanced techniques to estimate the service life of structures, cracks or crack like defects may occur due to several effects which may not be taken into account, such as underestimated external loads, mechanical vibrations, corrosive environment, collisions, fatigue etc. That is why there is a great number of studies on non-destructive testing (NDT) methods of structures. For local crack detection, it is well-known that the NDT methods such as x-ray, lamb waves, acoustic emission, etc. can be used. Nevertheless, these methods require a very hard work to examine the entire structure, inherently. For that reason, different methods which measure, and/or are based on a feature that is related to the whole structure gained a considerable attention. The natural frequencies, and corresponding mode shapes are unique for a structure since they contain information related to the loading and boundary conditions as well as the material and geometric properties of the structure. There are many works devoted to crack detection in beam–like structures using the modal properties. A comprehensive review of the literature on this subject is not within the scope of this study, but information about an enough number of studies for an overview is summarized below.

Existence of a crack on an elastic beam causes a local flexibility due to the strain energy concentration. Irwin (1957) came up with the idea of modeling the strain concentration by using an equivalent spring. This idea led to the development of a more general factor which is called the stress intensity factor (SIF) (Papaeconomou and Dimarogonas, 1989). More information about SIFs, and their expressions for different loading conditions and geometries can be found in the work of Tada et al. (1973). The effects of the crack on the global behavior of the structure are modelled by different approaches such as smaller cross–section (Kirshmer, 1944), or a link element (Harrison, 1973). In the following years, a more convenient way to model the compliance due to the crack is implemented. The additional flexibility is

* Corresponding author. Fax: +90 212 245 07 95.
E-mail addresses: ueroglu@itu.edu.tr (U. Eroglu), tufekcie@itu.edu.tr (E. Tufekci).

http://dx.doi.org/10.1016/j.ijsolstr.2016.06.005
0020-7683/

modelled by a spring, and stability, and vibrations of cracked beams are examined by many authors, such as Dimarogonas (1976), Chondors and Dimarogonas (1979), Anyfantis and Dimarogonas (1983), Ostachowicz and Krawwczuk (1992), and this idea is still being used widely. Reviews of especially the early stages of this topic can be found in the articles of Dimarogonas (1996), and Salawu (1997). One of the first studies about crack detection is the one by Adams et al. (1978), where changes in the axial vibrations of a bar with uniform or tapered cross–section are examined to detect the damage in such structures. Since the coupling between axial and transverse vibrations are not taken into account, the formulation was more accurate for small, or symmetric cracks. Transverse vibrations of a simply supported beam are used for the same purpose by Chang and Petroski (1986). Qian et al. (1990) used a finite element model to determine the natural frequencies of a cantilevered beam, taking the effect of crack closure into account. Also, a method based on the relationship between the crack, and modal parameters to determine the crack position from known natural frequencies is developed. Similar studies can be found in refs. Rizos et al. (1990); Pandey et al. (1991); Liang et al. (1991), and many more. A detailed review of this subject can be found in Salawu (1997), for example. With the development of the computer science, different methods to solve the dynamic problem of a cracked beam, such as finite element method (FEM), and to solve the inverse problem (i.e. detecting crack location and depth using modal properties) are implemented. He et al. (2001) implemented the genetic algorithms (GA) to solve the inverse problem. FEM is used to model the crack shaft, and the shaft is assumed to have one surface crack. They concluded that the numerical problems encountered in the solution of the inverse problem are eliminated with using GA. Saridakis et al. (2008) approximated the analytical model of a shaft with two cracks using neural networks. Only the bending modes are considered. GA is used to solve the inverse problem. It is concluded that their method reduces the computational time, and, therefore may be considered as an efficient tool for real – time crack detection. Xiang et al. (2008) used wavelet–based finite elements in modelling the cracked shaft. Rotatory inertia, and shear deformation is taken into account. They implemented a GA procedure to solve the inverse problem, by considering the first three frequencies. Formulation is validated through numerical examples as well as an experimental study. Another study which uses GA for crack detection is the one by Vakil-Baghmisheh et al. (2008). An analytical model without shear deformation and rotatory inertia effects is used to represent the dynamic behavior of a cracked beam. A detailed explanation about the GA is also presented in their study. Additionally, an experimental study is performed in order to validate the model. Khaji et al. (2009) proposed an analytical method to analyze cracked Timoshenko beams for crack detection. Mehrjoo et al. (2013) presented a new cracked finite beam element formulation based on the conjugate beam concept, and Betti's theorem. Shear deformation or rotatory inertia effects are not considered in the beam model, while crack is modelled as a linear spring. The inverse problem is solved using GA. Numerical examples are presented in order to validate the method. Caddemi and Morassi (2013) presented the exact closed form solutions of static and dynamic problems of multi cracked Euler-Bernoulli beams. Dirac's delta function, and rotational spring are used to model the flexibility due to a crack, and a justification of rotational spring model is proposed. Caddemi et al. (2013) presented the explicit expressions of stiffness matrix, mass matrix, and element shape functions of their FE model which is constructed to examine the dynamic behavior of damaged frame structures. Heaviside function is used to include the effect of discontinuity of the cross-section or the material, while discontinuities in transverse deflection, and slope are modelled by Dirac's delta function. Using these functions leads to the differential equation system of cracked beams, which does not require any additional transition conditions. The advantages of such a method are presented in the corresponding study. Mehrjoo et al. (2014) constructed a cracked finite beam element analogous to their earlier work (Mehrjoo et al., 2013), but used Timoshenko beam theory. GA is utilized in crack detection problem. Caddemi and Calio (2014) pursued the exact solution of the crack detection problem using the closed form expressions of vibration mode shapes, which are derived in the same work. Hakim et al. (2015) applied artificial neural networks to detect possible cracks in I-beams. For the training of the neural networks, results of 3D finite element simulations obtained from a commercial package are used. It is concluded that the method provides good results for even extra – light damages, which is possibly due to the fact that the first 5 natural frequencies are used in the inverse problem, unlike the majority of studies. Moezi et al. (2015) considered cantilever an Euler–Bernoulli beam with a crack. To estimate the crack location and depth, they applied a modified cuckoo optimization algorithm. It is concluded that modified cuckoo algorithm yields better results than cuckoo, and GA–Nelder-Mead algorithms. For other studies dealing with crack detection in beam–like structures, (see Chinchalkar et al., 2001; Patil and Maiti, 2003; Moradi et al., 2011; Mazanoglu and Sabuncu, 2012; Vakil-Baghmisheh et al., 2012; Surace et al., 2013; Pokale and Gupta, 2014; Cao et al., 2014; Rubio et al., 2015 for example).

The literature survey shows that the majority of the studies are focused on different techniques to solve the inverse problem, rather than a better modelling of the structure itself. It is also seen that most of the studies consider only the first three natural frequencies in order to detect cracks or crack-like defects. Even if the optimization algorithm used in the inverse problem may affect the computational time, the accuracy of the results is mainly based on the mathematical model of the structure. More realistic models yield better estimations of the natural frequencies, which gains more importance if one wants to consider higher modes in the solution of the inverse problem, and wants more precision. On the other hand, practicality of the mathematical formulation should be considered. Solution of the equations to get the results should not take too much time since hundreds of analysis might be needed to solve the inverse problem. In this respect, FEM looks to be a convenient method to model the beam.

In this study, an FEM formulation obtained by the exact solution of the beam equations is used for a better modelling of the structure. Only the in–plane motion of the beam is considered, and shear deformations, axial deformations, and rotatory inertia are included in the formulation. Crack is modelled as a rotational, and two translational springs. By using the transition conditions in the analytical modelling, a cracked beam finite element formulation is obtained. The formulation is also adopted to the frame structures. GA is used to solve the inverse problem. Proposed method is validated through numerous examples existing in the literature, as well as an experimental study.

2. Analysis

2.1. Exact solution of the beam equations

Assuming the cross–section of the beam shown in Fig. 1 is rigid, and symmetric about *y-z* plane, differential equation system of the static behavior is known as,

$$\frac{\mathrm{d}w}{\mathrm{d}z} = \frac{F_z}{EA}$$
$$\frac{\mathrm{d}u}{\mathrm{d}z} = -\Omega_x + \frac{k_y F_y}{GA}$$