International Journal of Solids and Structures 92–93 (2016) 64–75

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Multiphysics analysis of an asymptotically correct piezoelectric sensor under static and dynamic load

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ARTICLE INFO

Article history:
Received 8 December 2015
Revised 14 April 2016
Available online 26 April 2016

Keywords:
Piezoelectric
Variational asymptotic method
Sensor
Recovery

ABSTRACT

This paper presents a versatile platform named Variational Asymptotic Method (VAM), to model and analyze a piezoelectric cantilever sensor under static and dynamic load. Variational asymptotic method is a mathematically rigorous dimentional reduction methodology and has been previously used to model different structures which can be defined by an energy functional having one or more small inherent parameters; but, it has never been explicitly applied to capture the multiphysics behavior of a sensor. Piezoelectric based sensing technology has seen an explosive growth in the last decade with its various applications in different domain, such as energy harvesters, aerospace application, soft robotics, wind turbine, biomechanics, etc. So, an efficient mathematical model is highly required to get a better insight of the multiphysics behavior of an electromechanical structure. The present study highlights the capability of the theory to effectively capture the electromechanical response of a piezo-sensor under any static or dynamic load. The example problems considered in this present study include a single layer piezo-sensor as well as a double layer piezo-sensor bonded with aluminum. For static analysis, we have testified the model under a constant tip load. The discussed method effectively captures the voltage distribution across the thickness of the structure which is one of the fundamental parameter for a sensor model. The response accuracy obtained through variational asymptotic method is very good compared with the 3D simulation response performed in ABAQUS. For dynamic analysis a single layered piezo-sensor has been studied under a tip harmonic force. The structure response is studied for both damped as well as undamped conditions. Displacement and corresponding voltage output for specific excitation frequency which is close to its first natural frequency has been studied. The model efficacy under dynamic load has been validated with the experimental study performed by Ly et al. (2011). The present model has numerous potential applications like in PZT cantilever design for chemo-sensing, disease diagnostic, energy harvesting for self powered electronics, AFM higher harmonic imaging etc. The present theory along with the piezoelectric physics have been implemented in a modified version of VABS II (2004).

1. Introduction

A piezoelectric sensor can be defined as “A device that uses piezoelectric effect, to measure changes in pressure, acceleration, temperature, strain or force by converting them to electrical charge or voltage”. Piezoelectric effect was first discovered by Pierre Curie in the year 1880, and only in the late 1950s piezoelectric effect started to be used for industrial sensing applications. Piezoelectric sensors converts mechanical energy into electrical energy, which can be stored and used to power other devices as well, like self powered electronics (Sodano and Inman, 2005; Ng and Liao, 2005). The reduction in power requirements of small electronics has motivated many researchers to use piezoelectric sensors as alternative power source, which has found various application in remote and wireless sensing technology. Williams and Yates (1996) proposed the three very basic sensing mechanisms, i.e. electromagnetic, piezoelectric and electrostatic. Among these three sources, piezoelectric has proved to be the most prominent one. Chee et al. (2006) has reviewed on the mathematical models that were proposed on piezoelectric transduction. Yang and Lee (1994) showed some of the early works on structures with piezoelectric layers which included a stepped beam model that can predict analytically the natural frequencies and mode shapes at different piezoelectric sensor actuator locations. Similarly, Shen (1994) developed a one dimensional theory for modeling and analysis of a beam with piezoelectric actuators and sensors. Wang and Quek (2002) proposed an Euler beam model with embedded piezoelectric layers and have discussed two ways of connecting the electrodes on the

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http://dx.doi.org/10.1016/j.ijsolstr.2016.04.025
0020-7683/

surfaces of the piezoelectric layers. A digital regulator was designed and experimentally implemented for a beam containing piezoelectric sensor and actuators by Abreu Gustavo et al. (2003). The procedure as mentioned in the paper for placing the sensors and actuators along the smart structure found to be very effective. Experiments were also performed which demonstrates the effectiveness of the developed controller in reducing the vibration of the fexible beam. A FE-model for static and dynamic analysis of piezoelectric bimorph was discussed by Wang (2004). It combines 2D single layer representation model for mechanical displacement field and layer wise like approximation for the electric potential field. A PVDF bimorph beam and PZT plate were used to verify the present model. Kapuria and Yasin (2010) proposed an efficient FE-model for active vibration control of smart laminated beams integrated with electroded piezoelectric sensors and actuators. Kapuria's model discussed a beam element with two conventional nodes and one electric mode. The directional actuation and sensing using piezoelectric fiber reinforced composite (PFRC) actuators and sensors in active vibration suppression was examined for smart fiber metal laminate plate (Kapuria and Yasin, 2013). Zhou et al. (2005) presented an analytical model of piezoelectric multilayer cantilever as a micro chemical sensor. The voltage output from the model is in millivolt range and can be an appropriate alternative to the conventional laser based position sensitive detection system. There are wide field of application of piezoelectric sensors such as bitmap charge mode force sensor (Kursu et al., 2009), strain sensor based on piezoelectric paint film with its major applications in structural vibration monitoring (Payo and Hale, 2011), MEMS silicon based AC current sensor (Olszewski et al., 2014) and energy harvesters based on bending mode (Ly et al., 2011) and shear mode (Zhoua et al., 2012). A muscle contracting sensing system was presented by Han and Kim (2013) where using piezoelectric probes a resonance based active muscle stiffness sensor was developed to measure stiffness change in muscles. Recently, Buxi et al. (2014) investigated a frequency sensing circuit for measuring low level medical vibrations from ceramic piezoelectric sensor. Kalantarian et al. (2015) developed a necklace based piezoelectric sensor to monitor eating habits which is very crucial for maintaining a healthy lifestyle. Piezoelectric materials are distinguished mainly on the remarkable property to create a conversion interface between two forms of energy, i.e. mechanical and electrical energy. Ajitsaria et al. (2007) and Dalessandro and Rosato (2005), modeled a piezoelectric bender for voltage generation and studied the dynamic response. The modeling of the intelligent (piezoelectric) structures can be categorized in terms of structural configurations (Ali et al., 2004) and also according to the type of modeling. Mathematical modeling is the key element in the design process to understand various interrelated parameters. Any dimensionally reduced structure can be mathematically modeled by three basic approaches. Firstly, Newtonian approach which is based on force and moment balance, secondly asymptotic approach based on the asymptotic expansion of the 3D terms and thirdly variational asymptotic method (VAM) which includes the merits of varitaional approach along with asymptotic approximations of the 3D functional without any ad hoc or previous kinematic assumptions. Variational asymptotic method was first introduced by Berdichevsky (1979). The methodology works with the merits of both variational methods as well as asymptotic methods. One of the essential requirements of VAM based analysis is the existence of the small parameters, and a governing functional, whose extremum leads to the correct solution of the problem. In short, VAM can be termed as a rigorous mathematical tool applicable to any problem governed by an energy functional having one or more small parameter like slenderness for a beam like sensor model which we are trying to analyze here. Yu and Hodges (2004) have discussed how variational asymptotic method (VAM) has a strong mathematical foundation which is able to reproduce numerically the same result for a beam as one obtains from three dimensional elasticity but with less computational cost. VAM was successfully used to model composite beam (Yu, 2002) as well as smart beams with piezoelectric fiber composites (Roy et al., 2007; Neto et al., 2009). Recently a fully coupled rod model was developed by Roy and Yu (2009). Yu et al. (2012) also discusses three updates in the theory for a composite beam model. But the direct piezoelectric effect, i.e. the sensory effect has not yet been analyzed using this theory.

In this present study a sensor has been analyzed as a slender beam model with cantilever boundary condition. The methodology works on by taking advantage of the slenderness of the structure and asymptotically splitting the original 3D problem into a 2D coupled cross-sectional analysis and a 1D beam analysis. The cross-sectional analysis has been implemented using finite element in the computer code VABS, which when combined with 1D beam analysis provide an asymptotically correct 1D constitutive model for smart beams without special assumptions regarding the geometry and the material of the cross-section, distribution of the electric field, the location of the smart materials such as embedded or surface mounted. With the 1D constitutive model and the recovery relations, we can get the full 3D responses of the sensor model. The validation of mathematically rigorous 3D recovery module for a sensor is another key contribution of the present study. Thus variational asymptotic method is a valid methodology which can be used to avoid the difficulties in dealing with the 3D elasticity solution for the same problem. Some standard sensor examples have been studied thoroughly like a cantilever beam with piezoelectric material bonded with aluminium and subjected to external static load as well as a sensor under time varying load, validated with the 3D simulation results obtained from Abaqus/CAE 6.10, Dassault system (2010) and with experimental results (Ly et al., 2011) to prove the accuracy and efficacy of the present theory for piezoelectric sensing. The excellent agreement between the present theory and ABAQUS 3D multiphysics simulation as well as with experimental results demonstrates that one can use this mathematical model to greatly simplify the overall procedure without significant loss in accuracy.

2. Methodology

Fig. 1 projects an overview of the methodology used to develop the mathematical model of sensor. The rectangles represent the analysis and the ellipses represents the bodies of input and output data. The geometry and the material properties of the piezoelectric cantilever beam are the input to the 2D cross-sectional analysis. The 2D cross-sectional analysis is performed by VABS (Variational asymptotic beam sectional analysis) written in FORTRAN 90. Cesnik and Hodges (1997) provides a seminal work in developing VABS. The work done by Hodges (2006) in his book Nonlinear Composite Beam Theory and Yu (2002) in his Ph.d. work, provides the background and theory for VABS. Later Roy (2007) during his Phd work extended the theory for electromechanical structures made up of piezoelectric material. Thus, VABS used for this current work is a modified version of VABS II released in the year 2004 (Yu, 2011) with an addition of piezoelectric physics.

3. Kinematics of the structure

The sensor model in our present study has been analyzed as a slender beam made up of piezoelectric material. So we will try to analyze a beam and its electromechanical behavior. Kinematics deals with the relationship between strains and displacements. The main feature of the sensor model consists of mechanical and electrical fields. The mechanical sub-divison comprises of the 3D strain