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Effects of surface tension on the adhesive contact between a hard sphere and a soft substrate

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ABSTRACT

In the present paper, the adhesive contact between a rigid sphere and a compliant elastic substrate with surface tension is analyzed. Using the minimum potential energy principle and considering both the effects of surface tension and adhesion, explicit solutions are obtained for the contact radius and the indent depth for the case without external loading. It is found that surface tension evidently alters the pressure distribution in the contact region and tends to decrease both the contact radius and the indent depth. The present model establishes a bridge between the JKR model for adhesive contact problems and the Young-Laplace's law of surface tension.

1. Introduction

Contacts between solid surfaces are not only ubiquitous in nature but also a major concern in engineering. The classical Hertzian solution for non-adhesive elastic contact problems assumes that the interaction in the whole contact region is compressive (Hertz, 1882). However, when two solid surfaces are brought into close proximity, such molecular interactions as van der Waals forces may lead to attractive adhesion between the two surfaces. Bradley (1932) adopted the Lennard-Jones potential to describe the adhesive force between two rigid spheres. Through the balance between the stored elastic strain energy and surface energy, Johnson et al. (1971) formulated the well-known JKR adhesive contact model, which predicts infinite tensile traction at the contact edge. Derjaguin et al. (1975) presented an alternative adhesive contact model, in which the contact profile remains the same as that in the Hertzian model and an additional attractive force acts outside the contact region. Using the Dugdale model to characterize the attractive force, Maugis (1992) developed a more general theory to describe the transition between the JKR and DMT models.

The JKR model has been the fundament for the analysis of contact problems at micro and nano scales. However, the results of the JKR model have a distinct deviation from the recent experiments of the adhesive contact between micro/nano-sized hard particles and soft substrates (Rimai et al., 2000; Chakrabarti and Chaudhury, 2013). For sufficiently small particles and soft substrates, Style et al. (2013) reported that the adhesion mimics the adsorption of particles on a liquid surface due to surface tension.

To address the influence of surface stress in solids, Gurtin and Murdoch (1975) and Gurtin et al. (1998) established a surface elasticity theory. This theory has been used in micromechanics to analyze the elastic field around nanosized inhomogeneities (Sharma et al., 2003; Lim et al., 2006) and the effective elastic moduli of nano-composites (Duan et al., 2005; Gao et al., 2006). Huang and Wang (2006) formulated the theory of surface elasticity at finite deformation, and then Huang and Sun (2007) gave a linear version of it and applied the linear theory to study the effective elastic constants of nano-composites. Their work is advantageous to address the effect of residual surface stress, which has long been ignored.

The influence of surface tension on non-adhesive contact has attracted considerable attention in recent years. To evaluate the elastic modulus of inflated lobes of lung, Hajji (1978) studied the axisymmetric indentation on an elastic half space with a pre-stressed membrane. Using the surface elasticity theory, He and Lim (2006) and Huang and Yu (2007) derived the three- and two-dimensional surface Green's functions, respectively. Through the Fourier integral transformation method, Wang and Feng (2007) solved the elastic field induced by a concentrated force acting on a half plane with surface tension. Using Stroh's formulism, Koguchi (2008) obtained the surface Green's function for an anisotropic half space with surface effects. Chen and Zhang (2010) presented the surface Green's function for anti-plane shear deformation. Gao et al. (2013) investigated the Boussinesq problem with both surface tension and surface elasticity, showing a dominant influence of surface tension over surface elasticity under normal loading.

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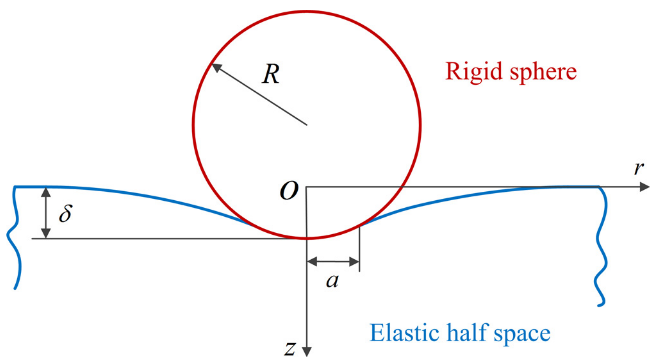

Fig. 1. Adhesive contact between a rigid sphere and an elastic half space.

Recently, the effects of surface tension on adhesive contact have also been addressed. Salez et al. (2013) studied the adhesion of a spherical elastic particle on a rigid substrate and established a bridge between the JKR and Young-Dupre asymptotic regimes. Gao et al. (2014) analyzed the surface effects on adhesive contact by assuming that the pressure within the contact region takes the same form as that in the JKR model. Using the finite element method with the incorporation of surface tension, Xu et al. (2014) addressed the adhesion between a rigid sphere and a compliant elastic half space under finite deformation. Recently, Hui et al. (2015) studied the influence of surface tension on the no-slip adhesive contact between a rigid sphere and an incompressible isotropic elastic substrate. In this significant work, the energy release rate in fracture mechanics was adopted and calculated by the conventional compliance approach. They gave the contact radius and the indent depth in an implicit form. This problem is reconsidered in the present paper in order to give a simple and explicit solution. By using the minimum potential energy principle, we present a straightforward method to solve the adhesive contact problem with surface tension. The explicit expressions of the contact radius and the indent depth are given, which are more convenient in practical applications.

2. Adhesive contact with surface tension

Consider the adhesive contact between a rigid sphere with radius R and an isotropic elastic half space, as shown in Fig. 1. Refer to the Cartesian coordinate system (r, z), where the origin O is located at the initial contact point, the r axis along the initial surface of the half space, and the z axis perpendicular to the surface. In the absence of external load, the contact is driven by the adhesion between the rigid sphere and the elastic half space, and resisted by surface tension and the strain energy. Let δ denote the indent depth and a the contact radius in the equilibrium state.

For the considered axisymmetric problem, we express the pressure within the contact region as $p(t)$, where t is the distance from the origin. By using the solution of a point load acting on an elastic half space with surface tension (Hajji, 1978), the displacement boundary condition within the contact region can be described by an integral equation

$$\frac{1}{2\tau^0}\int_0^a\int_0^\pi\left[H_0\left(\frac{l}{s}\right)-Y_0\left(\frac{l}{s}\right)\right]tp(t)\,\mathrm{d}t\mathrm{d}\theta=\delta-\frac{r^2}{2R},\tag{1}$$

where H_n and Y_n are the Struve function and Bessel function of the second kind of order n, $l=(r^2+t^2-2rt\cos\theta)^{1/2}$, τ^0 is the substrate-air surface tension, E^* is the composite elastic modulus of the elastic half space, and

$$s=\frac{2\tau^0}{E^*},\tag{2}$$

is an intrinsic material length indicating the influencing scope of surface tension (Long and Wang, 2013).

In the absence of external load, the resultant force within the contact region must vanish, that is,

$$2\pi\int_0^a tp(t)\mathrm{d}t=0.\tag{3}$$

Differentiating Eq. (1) with respect to r, one has

$$\int_0^a\int_0^\pi\left[H_1\left(\frac{l}{s}\right)-Y_1\left(\frac{l}{s}\right)-\frac{2}{\pi}\right]\frac{r-t\cos\theta}{l}tp(t)\,\mathrm{d}t\mathrm{d}\theta=2\tau^0 s\frac{r}{R}.\tag{4}$$

Letting r=0 in Eq. (1), the indent depth is obtained as

$$\delta=\frac{\pi}{2\tau^0}\int_0^a\left[H_0\left(\frac{t}{s}\right)-Y_0\left(\frac{t}{s}\right)\right]tp(t)\mathrm{d}t.\tag{5}$$

The elastic energy stored in the elastic half space can be calculated by the contact pressure

$$U_{\mathrm{el}}=\frac{1}{2}\int_A p(t)\bar{u}_z(t)\mathrm{d}S=\pi\int_0^a\left(\delta-\frac{t^2}{2R}\right)tp(t)\mathrm{d}t,\tag{6}$$

where A denotes the contact region and $\bar{u}_z$ is the normal displacement on the surface of the half space.

Eqs. (4) and (3) can be normalized into the following forms

$$\frac{1}{\pi}\int_{-1}^1\mathrm{kern}(r',t')q(t')\frac{\mathrm{d}t'}{\sqrt{1-t'^2}}=\frac{2s'}{\pi}(r'+1),\tag{7}$$

$$\frac{1}{\pi}\int_{-1}^1 q(t')\frac{\mathrm{d}t'}{\sqrt{1-t'^2}}=0,\tag{8}$$

respectively, where

$$q(t')=\frac{R}{\tau^0}\sqrt{1-t'^2}(t'+1)p\left(\frac{a}{2}(t'+1)\right),\tag{9}$$

$$\mathrm{kern}(r',t')=\int_0^\pi\left[H_1\left(\frac{l'}{s'}\right)-Y_1\left(\frac{l'}{s'}\right)-\frac{2}{\pi}\right]\times\frac{(r'+1)-(t'+1)\cos\theta}{l'}\mathrm{d}\theta,\tag{10}$$

$$r'=\frac{2r}{a}-1,\quad t'=\frac{2t}{a}-1,$$
$$l'=\frac{2l}{a}=[(r'+1)^2+(t'+1)^2-2(r'+1)(t'+1)\cos\theta]^{1/2},\tag{11}$$

$$s'=\frac{2s}{a}=\frac{4\tau^0}{E^*a}.\tag{12}$$

Using the Gauss-Chebyshev quadrature formula, Eqs. (7) and (8) can be written in the matrix form as (Erdogan and Gupta, 1972)

$$\mathbf{BQ}=\mathbf{F},\tag{13}$$

where

$$\mathbf{B}=[b_{ij}],\quad b_{ij}=\frac{1}{n}\mathrm{kern}(r'_i,t'_j),\quad b_{nj}=\frac{1}{n},$$
$$\mathbf{Q}=[q(t'_1),q(t'_2),\ldots,q(t'_n)]^T,$$
$$\mathbf{F}=\frac{2s'}{\pi}[r'_1+1,r'_2+1,\ldots r'_{n-1}+1,0]^T,$$
$$r'_i=\cos\frac{i\pi}{n},\quad t'_j=\cos\frac{(2j-1)\pi}{2n},\quad i=1,2,\ldots n-1,$$
$$j=1,2,\ldots n.\tag{14}$$

For a given value of contact radius a (or s'), one can calculate the corresponding values of $q(t'_j)$ by solving the linear