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Analysis of thermoelastic stress-concentration around oblate cavities in three-dimensional generally anisotropic bodies by the boundary element method

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ABSTRACT

In this article, stress concentrations around cavities in 3D generally anisotropic bodies are investigated using the boundary element method (BEM), where the associated volume integral is analytically transformed to the boundary. All derivations are based upon the Fourier-series representations of the fundamental solutions, including Green's function of displacements and its derivatives. This approach of analytical transformation has fully restored the BEM's distinctive notion that only the boundary needs to be discretized. The goal of the present work is to investigate the thermoelastic stress-concentration around cavities in 3D anisotropic bodies by use of the analytically transformed boundary integral equation (BIE). The work has fully recovered the BEM's nature of boundary discretization for treating 3D generally anisotropic thermoelasticity. This is the first implemented work that successfully treats thermoelastic problems for 3D generally anisotropic solids by the analytically transformed BIE. In the paper, Interesting phenomena are observed from the analyses of stress concentrations around oblate cavities and some discussions are made.

1. Introduction

In engineering practice, stress concentrations around cavities often play a crucial role for the integrity of structures. In various applications, anisotropic materials with cavities are often subjected to thermal loads, resulting stress concentrations around the cavities. For assessing the integrity of such applications, investigation of thermoelastic stress concentrations around cavities in anisotropic bodies appears to be an important topic.

Generally speaking, investigation of the phenomena can be performed by experiments, analytical study, or numerical analysis. Since there are too many works within this broad scope to cover for a thorough review, only a few among them for the steady-state are mentioned here as examples. Some early works can be referred to Goodier and Florence (1959) and Florence and Goodier (1964), focusing on the localized thermal stress at holes, cavities and inclusions due to the disturbance of a uniform heat flow. Some others (e.g. Hoffman and Ariman, 1970; Rao et al., 1971; Matsumoto and Sekiya, 1982) have studied the thermal stresses in thin elastic finite plates with insulated circular, elliptic, and rectangular holes. Chao and Gao (2001) have studied the mixed boundary-value problems of two-dimensional anisotropic thermoelasticity with elliptic boundaries. Despite its importance for fundamental studies, analytical analysis is only applicable to problems with simple geometry and boundary conditions, especially for two-dimensional cases. For problems with complicated boundaries in general, the analysis will need numerical tools, such as the finite element method (FEM) or the boundary element method (BEM).

In contrast with the domain solution techniques, the BEM is well recognized as an efficient alternative to the more commonly used FEM being a tool for engineering analysis. For linear elastic solids, the analytical basis of the method is the boundary integral equation (BIE) that relates the displacements and tractions on domain surface when thermal loads and body-force are absent. This characterizes the BEM's distinctive feature, namely only surface modeling of the domain involved. In comparison with the domain solution techniques, this boundary-discretized feature is even more advantageous for analyzing stresses with rapid variations near the cavity surface in anisotropic bodies. However, in the direct formulation of BIE, thermal effects reveal themselves as a volume integral that conventionally requires "cell-discretization" throughout the whole domain for its numerical integration. Apparently, such a treatment not only is inefficient for numerical integrations but also destroys the BEM's distinctive notion that only the boundary needs to be modeled.

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To avoid direct integration of this volume integral, several techniques have been proposed over the years, including the dual reciprocity method (e.g. Nardini and Brebbia, 1982), multiply reciprocity method (e.g. Nowak and Brebbia, 1989), particular integral approach (e.g. Deb and Barnerjee, 1990), and the exact transformation method (ETM) (e.g. Rizzo and Shippy, 1977, among others). Among these schemes, the ETM is the most appealing since it restores the analysis to a purely boundary one without invoking additional simplifications and/or numerical approximations unlike the others. For isotropic thermoelasticity under steady state, this volume integral can be exactly transformed to the boundary by the ETM in both 2D and 3D (e.g. Rizzo and Shippy, 1977; Danson, 1983). However, extension of such transformation towards the same end in the BEM analysis for 3D generally anisotropic thermoelasticity has remained challenging due to the mathematical complexity of the associated fundamental solutions.

Over the past several decades, the topic of evaluating the fundamental solutions for 3D anisotropic elastic bodies has remained a focus of investigation. In general, the methods for evaluation of the Green's function include the numerical integration method (e.g. Fredholm 1900; Barnett, 1971; Wang, 1997; Wang and Denda, 2007), residue calculus method (e.g. Ting and Lee, 1997; Phan et al., 2004, 2005; Lee, 2003, 2009; Buroni and Sáez, 2013), and the Stroh formalism method.

For simplifying the evaluation process, Shiah (2014) employed an approach re-expressing the fundamental solutions into the forms of double Fourier series to study the inertial effects in 3D anisotropic solids. Following the success of implementing the double-Fourier-series forms to BEM analysis, Shiah and Tan (2014) found that the series expressions were very ideal to be applied to deal with thermoelastic effects, too; however, no implementation of that work was achieved at that time. In the present work, this approach is to further extend the author's previous work (2014) to study the thermoelastic stress-concentration around cavities in 3D generally anisotropic bodies. In the end, a few benchmark examples are presented.

2. Anisotropic thermoelasticity

For a generally anisotropic elastic solid, the constitutive relationship between the stress σ_{ij} and the strain ε_{ij} when temperature change Θ is also considered, is governed by the well-known Duhamel–Neumann relation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - \gamma_{ij}\Theta, \quad (i, j, k, l = 1, 2, 3), \tag{1}$$

where c_{ijkl} and γ_{ij} denote the elastic constants (stiffness coefficients) and thermal modulii, respectively, of the material. The stiffness coefficients $\mathbf{C}$ to be used for our analysis are arranged in the order according to

$$\sigma = (\sigma_{11},\ \sigma_{22},\ \sigma_{33},\ \sigma_{23},\ \sigma_{13},\ \sigma_{12})^{\mathrm{T}}, \tag{2}$$

$$\varepsilon = (\varepsilon_{11},\ \varepsilon_{22},\ \varepsilon_{33},\ 2\varepsilon_{23},\ 2\varepsilon_{13},\ 2\varepsilon_{12})^{\mathrm{T}}. \tag{3}$$

For a generally anisotropic body, the thermal modulii in Eq. (1) are given by

$$\gamma_{ij} = C_{ijkl}\,\alpha_{kl}, \tag{4}$$

where α_{kl} stands for the coefficients of thermal expansion. As in the usual manner to treat steady-state sequentially coupled thermoelasticity, the resulting elastic field is determined from the temperature distribution corresponding to the boundary conditions prescribed for heat conduction analysis. For this, the thermal field can be solved independently but must be first obtained before solving the elastostatic problem.

Under the condition of steady state without heat source, the anisotropic heat conduction is governed in the Cartesian coordinate system by

$$K_{11}\frac{\partial^2\Theta}{\partial x_1^2} + K_{22}\frac{\partial^2\Theta}{\partial x_2^2} + K_{33}\frac{\partial^2\Theta}{\partial x_3^2} + 2K_{12}\frac{\partial^2\Theta}{\partial x_1\partial x_2} + 2K_{13}\frac{\partial^2\Theta}{\partial x_1\partial x_3} + 2K_{23}\frac{\partial^2\Theta}{\partial x_2\partial x_3} = 0\,, \tag{5}$$

where K_{ij} are the conductivity coefficients.

Eq. (5) can be transformed to its canonical form of the Laplace equation by a simple coordinate transformation (Shiah and Tan, 2014),

$$\hat{\mathbf{x}}^T = \mathbf{F}\,\mathbf{x}^T, \tag{6}$$

where $\hat{\mathbf{x}}$ and $\mathbf{x}$ represent the transformed and the original coordinates, respectively; $\mathbf{F}$ denotes the transformation matrix given by

$$\mathbf{F} = \begin{pmatrix} \sqrt{\Delta}/K_{11} & 0 & 0 \\ -K_{12}/K_{11} & 1 & 0 \\ \beta_1 & \beta_2 & \beta_3 \end{pmatrix}, \tag{12}$$

where all the coefficients are defined by

$$\Delta = K_{11}K_{22} - K_{12}{}^2, \tag{13a}$$

$$\beta_1 = (K_{12}K_{23} - K_{13}K_{22})/\sqrt{\omega}, \tag{13b}$$

$$\beta_2 = (K_{12}K_{13} - K_{23}K_{11})/\sqrt{\omega}, \tag{13c}$$

$$\beta_3 = \Delta/\sqrt{\omega}, \tag{13d}$$

$$\omega = K_{11}K_{13}\Delta - K_{11}K_{12}K_{13}^2 + K_{11}K_{12}K_{13}K_{23} - K_{23}^2K_{11}^2. \tag{13e}$$

By the coordinate transformation, the original heat conduction is now governed by the standard Laplace equation in the transformed coordinate system as denoted by the underscore, i.e.

$$\Theta_{,\underline{ii}} = 0. \tag{14}$$

As a result, the thermal field can be determined using the standard boundary integral equation for the potential theory. Once the temperature field in the body is determined via solving the BIE for the mapped domain, the solution for the corresponding elastic field of the solid body can then proceed.

3. The BIE for anisotropic thermoelasticity

As has been well established in the literature for the direct BEM formulation, the displacements u_i and the tractions t_i at the source point P and the field point Q on the surface S of an elastic body are related by the following integral equation:

$$C_{ij}(P)\,u_i(P) + \int_S u_i(Q)T_{ij}^*(P,Q)dS = \int_S t_i(Q)U_{ij}^*(P,Q)dS + \int_V B_i(q)U_{ij}^*(P,q)\,d\Omega, \tag{15}$$

where q is an arbitrary field point inside the domain Ω; B_i denotes an effective body-force component due to the thermal and/or inertia effects. It is evident that unless the last integral on the right hand side of Eq. (15) is transformed to surface ones, its direct numerical evaluation will require interior discretization of the whole domain. Also in Eq. (15), U_{ij}^* and T_{ij}^* are the fundamental solutions for the displacements and tractions, respectively, in the ith direction at the field point due to a unit load in the jth direction at the load point. As is well known in solid mechanics, thermal effects can be treated as an equivalent body-force in the governing equations in elasticity. It can be easily established that Eq. (15) becomes