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Stretchability and compliance of freestanding serpentine-shaped ribbons

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ABSTRACT

High-performance stretchable electronics have to utilize high-quality inorganic electronic materials such as silicon, oxide or nitride dielectrics, and metals. These inorganic materials usually crack or yield at very small intrinsic strains, for example, 1%, whereas bio-integrated electronics are expected to at least match the stretchability of bio-tissues (20%) and deployable structure health monitoring networks are expected to expand from wafer scale (several centimeters) to cover macroscopic structures (several meters). To minimize strains in inorganic materials under large deformation, metallic and ceramic films can be patterned into serpentine-shaped ribbons. When the ribbon is stretched, some sections of the ribbon can rotate and/or buckle to accommodate the applied displacement, leaving much smaller intrinsic strain in the materials compared to the applied strain. The choice of the shape of the serpentine depends on systematic studies of the geometric variables. This paper investigates the effect of serpentine shapes on their stretchability and compliance through theoretical, numerical, and experimental means. Our closed-form curved beam solutions, FEM results, and experimental measurements have found good agreement with one another. Our results conclude that in general, the narrower ribbon, the larger arc radius and arc angle, and the longer arm length will yield lower intrinsic strain and effective stiffness. When the arm length approaches infinite, the stretchability can be enhanced by several orders. A few unexpected behaviors are found at arc angles that are close to straight bars. With additional practical constraints such as minimum ribbon width and finite overall breadth, the optimal serpentine shape can be accurately determined using our closed-form analytical solution.

1. Introduction

In recent years, stretchable electronics start to find exciting applications in wearable electronics, smart surgical tools, compliant power sources, as summarized in four recent review articles (Kim et al., 2012a,b,c; Lu and Kim, 2013). Examples include epidermal electronics for vital-sign monitoring (Kim et al., 2011; Huang et al., 2012; Yeo et al., 2013), fingertip electrotactile actuators for smart surgical gloves (Ying et al., 2012), instrumented balloon catheters for minimally invasive surgeries (Kim et al., 2011), multifunctional cardiac webs (Kim et al., 2012), and stretchable batteries (Xu et al., 2013). A fundamental challenge of stretchable electronics is to build a mechanically compliant system which is able to deform drastically without affecting its electrical functionalities. High-performance stretchable electronics have to utilize high-quality, inorganic electronic materials such as silicon, gold, and oxides or nitrides, which are known to be very stiff and even brittle materials. For example, thin films of ceramic materials such as silicon, oxides, and nitrides tend to rupture at very small strains, around 1% (Leterrier et al., 1997; Gleskova et al., 1999; Sun et al., 2012). Although copper thin films well bonded to polyimide substrates have been stretched beyond 50% without fracture (Lu et al., 2007), most of the deformation is plastic and therefore, irreversible. The elastic regime of metal deformation is still limited to 1% or less (Hommel and Kraft, 2001). To minimize strains in metal interconnects under large deformation, either out-of-plane sinusoidal nanoribbons/nanomembranes (Lacour et al., 2005; Khang et al., 2006) or in-plane serpentines (Gray et al., 2004; Kim et al., 2008) are employed to replace straight wires. Comparing the two popular strategies, in-plane serpentines are more fabrication and integration friendly because no pre-stretch of the rubber substrate is required and low profile is preserved. In fact, not only metals but also silicon and two-dimensional graphene can be patterned into

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serpentine ribbons to achieve high stretchability (Kim et al., 2011, 2008, 2011; Zhu et al., 2014). Recently developed micro-transfer-printing techniques (Meitl et al., 2006) has enabled the integration of well-patterned inorganic films and ribbons on elastomer substrates. When the substrate is stretched, serpentine ribbons can rotate in plane and buckle out of plane to accommodate the applied deformation, resulting in greatly reduced intrinsic strains in the inorganic materials as well as minimized stiffness at the system level. Low system-level effective stiffness is critical for bio-integration because devices as compliant as bio-tissues can conform to and deform together with the bio-tissue without detachment or imposing any significant mechanical constraint (Kim et al., 2011; Huang et al., 2012; Yeo et al., 2013).

Besides stretchable electronics, serpentine structures can also be found in a lot of expandable systems made out of intrinsically stiff materials. Examples include the cardiovascular stents for angioplasty (Beyar et al., 1997) or percutaneous coronary intervention (Roguin et al., 1997), and deployable sensor networks for structural health monitoring (Lanzara et al., 2010). Tubular metallic stents in the form of a serpentine-meshed scanfold can be inserted into blood vessels in a very small initial diameter, tunnel through the veins and arteries, and get expanded by more than 200% using a balloon catheter, to provide support inside the patient's arteries. As another example, ultra narrow, highly tortuous serpentines were incorporated in the design of a spider-web-like highly expandable sensor network (Lanzara et al., 2010). The microfabrication of inorganic-material-based sensors which has to utilize regular-sized spinners, mask aligners, and vacuum chambers can all be performed on a wafer-sized rigid substrate. Once the circuit is released from the rigid substrate, the sensor network can be deployed by more than 100 times in area so that they can cover macroscopic civil or aerospace structures to perform structure health monitoring. In both examples, the large expandability comes from just the in-plane rigid body rotation of the freestanding serpentine ligaments.

Although serpentines have been widely used as the stretchable configuration of stiff materials, the designs of the serpentine shape are still largely empirical. According to existing studies, the applied strain-to-rupture of metallic serpentine ribbons varies from 54% to 1600%, depending on the geometric parameters such as ribbon width, arc radius, arm length, substrate support, and so on (Gray et al., 2004; Kim et al., 2008; Lanzara et al., 2010; Hsu et al., 2009; Brosteaux et al., 2007; Xu et al., 2013). A few experimental and finite element modeling (FEM) studies have been conducted to provide insights into the shape-dependent mechanical behavior (Gray et al., 2004; Hsu et al., 2009; Li et al., 2005) of serpentine ribbons. Two recent theoretical articles provided viable routes to predict the stretchability of buckled serpentines (Zhang et al., 2013a) and self-similar serpentines (Zhang et al., 2013b), but the shapes of the unit cells are very limited. Moreover, the effective compliance of the serpentine structure and the shape optimization under practical constraints have been rarely discussed. This paper performs analytical, FEM, and experimental studies on freestanding serpentines with three systematically varied dimensionless geometric parameters. The closed-form plane strain solution can be applied to predict the stretchability and effective stiffness of numerous serpentine structures. The analytical solution can also be used to optimize the three dimensionless geometric parameters under one optimization goal – maximum stretchability, and two practical constraints – e.g. no material overlap and finite breath of the structure at a given ribbon width resolution.

This paper is organized as follows. Section 2 summarizes the analytical, FEM, and experimental approaches we use. Section 3 compares the analytical, FEM, and experimental results for systematically varied serpentine shapes. Section 4 demonstrates how to determine the optimal serpentine shape under certain practical constraints using the analytical solutions. Concluding remarks are provided in Section 5. In Appendix A, derivations based on elasticity theory (i.e. Airy's function) are provided. Appendix B derives the load–displacement relation using Castigliano's method (i.e. energy method) and the strain distribution using the Winkler curved beam (CB) theories.

2. Analytical, fem, and experimental approaches

It has been observed that when freestanding serpentine ribbons have large width-to-thickness ratio, they tend to buckle out-of-plane when subjected to end tensile displacements (Kim et al., 2011, 2008; Li et al., 2005) whereas when their width-to-thickness ratios are small, the deformation is completely in plane (Lanzara et al., 2010). Although general three-dimensional (3D) theories for curved thin rods are available (Love, 2011), out-of-plane buckling and post-buckling analysis for curved beams only yields analytical solutions for very limited shapes and loading conditions (Kang and Yoo, 1994). To initiate the theoretical analysis and optimization of free standing serpentine ribbons, we start with a two dimensional (2D) plane strain model, which suppresses the out-of-plane buckling deformation. A unit cell cut out of a one-directional periodic serpentine ribbon is depicted in Fig. 1A. The unit cell of a so-called "horseshoe" serpentine is composed of an arc joined end to end with its upside-down mirror image. Based on the conventional horseshoe shape, we add a linear "arm" section between the two arcs. The unit cell of this generic serpentine can be well defined by four geometric parameters: the ribbon width w, the arc radius R, the arc angle α, and the arm length l. The ribbon thickness t is assumed to be unit in the plane strain model. Hence the end-to-end distance S of a unit cell can be expressed by

$$S = 4\left(R\cos\alpha - \frac{l}{2}\sin\alpha\right) \tag{1}$$

When this unit cell is subjected to a tensile displacement u_0 at each end, the effective applied strain ε_{app} is defined as

$$\varepsilon_{app} = \frac{2u_0}{S} \tag{2}$$

Therefore a straight ribbon (i.e. $\alpha = -90^\circ$) of length S should have a uniform strain of ε_{app} if the end effects are neglected. Attributing to symmetry and anti-symmetry, a unit cell can ultimately be represented by a quarter cell with fixed boundary at the axis of symmetry and a displacement of $u_0/2$ at the end, as shown in Fig. 1B. The reaction force is named P in Fig. 1B. The end of the unit cell is considered free to rotate because we assume it is a unit cell cut out of a long, periodic serpentine ribbon whose boundary conditions (i.e. whether clamped or simply-supported) do not affect the unit cell.

In this problem, two mechanical behaviors of serpentines are of particular interest to us: the stretchability and the effective stiffness. *Stretchability* is defined as the critical applied strain beyond which the material of the serpentine ribbon will rupture and will be denoted by ε_{app}^{cr}. Therefore, if the failure criterion $\varepsilon_{max} = \varepsilon_{cr}$ is adopted, where ε_{max} and ε_{cr} represent the maximum tensile strain and the intrinsic strain-to-rupture of the material, respectively, the normalized maximum tensile strain in the serpentine, $\varepsilon_{max}/\varepsilon_{app}$, will govern the stretchability by

$$\varepsilon_{app}^{cr} = \frac{\varepsilon_{cr}}{\frac{\varepsilon_{\max}}{\varepsilon_{app}}} \tag{4}$$

Effective stiffness is defined as the ratio between the reaction force P and the effective displacement, $2u_0$. With Young's modulus E and Poisson's ratio ν, the stiffness of a plane strain straight ribbon