International Journal of Solids and Structures 53 (2015) 48–57

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Fracture analysis of an electrically conductive interface crack with a contact zone in a magnetoelectroelastic bimaterial system

P. Ma [a,b], R.K.L. Su [a,*], W.J. Feng [b]

[a] Department of Civil Engineering, The University of Hong Kong, Hong Kong, China
[b] Department of Engineering Mechanics, Shijiazhuang Tiedao University, Shijiazhuang 050043, PR China

ARTICLE INFO

Article history:
Received 3 March 2014
Received in revised form 8 October 2014
Available online 31 October 2014

Keywords:
Electrically conductive interface crack
Contact zone
Magnetoelectroelastic material
Field intensity factor

ABSTRACT

An electrically conductive interface crack with a contact zone in a magnetoelectroelastic (MEE) bimaterial system is considered. The bimaterial is polarized in the direction orthogonal to the crack faces and is loaded by remote tension and shear forces as well as electrical and magnetic fields parallel to the crack faces. It is assumed that the electrical field inside the crack faces is equal to zero and the magnetic quantities are continuous across the crack faces. Using special expressions of magnetoelectromechanical quantities via sectionally-analytic functions proposed in this paper, a combined Dirichlet–Riemann and Hilbert boundary value problem is formulated and solved analytically. Explicit analytical expressions for the characteristic mechanical, electrical and magnetic parameters are presented. A simple transcendental equation is derived for the determination of the contact zone length. Stress, electric field and magnetic field intensity factors and the contact zone length are found for various loading cases. A significant influence of the electric field on the contact zone length, stress and electric field intensity factors is observed. Magnetoelectrically permeable conditions in the crack region are also investigated and comparisons of different crack models are performed. Results presented in this paper should have potential applications to the design of multilayered magnetoelectroelastic (MEE) structures and devices.

1. Introduction

As new multifunctional materials, magnetoelectroelastic (MEE) materials have found increasing applications in electronic technology, ultrasound technology, intelligence projects, as well as in other advanced smart structures, owing to their special magnetoelectric coupling effect. However, MEE materials are usually brittle, possessing low fracture toughness and high imperfection sensitivity. These characteristics make the MEE devices susceptible to the formation of imperfections, such as cracks, during manufacture or service life and further lead to structural performance failure. Therefore, there has been tremendous interest in studying the fracture and failure behaviors of such materials (Zhou et al., 2004; Gao et al., 2004; Hu and Li, 2005; Feng and Su, 2006; Feng et al., 2007; Yong and Zhou, 2007; Wang et al., 2008; Li, 2001; Gao et al., 2003; Sih et al., 2003; Tian and Gabbert, 2005; Hu and Li, 2005; Zhou et al., 2007; Wang and Mai, 2007; Zhao and Fan, 2008; Singh et al., 2009; Hou et al., 2009; Chen, 2009; Zhong et al., 2009; Wang et al., 2010; Zhong and Zhang, 2010; Sladek et al., 2011, 2012; Zhao et al., 2013; Han and Pan, 2013). In engineering practice, layered structures are very common. In such structures, interface delamination is probable, which will result in interfacial cracks. This is the main reason behind structural failure. In the past couple of decades, many researchers have studied the interface crack problems of MEE materials (Gao and Noda, 2004; Li and Kardomateas, 2007; Zhao et al., 2008; Herrmann et al., 2010; Zhu et al., 2010; Feng et al., 2011; Ma et al., 2012).

On the other hand, electrically conductive cracks are very likely to form due to an extremely high local electric field inside the crack, since the dielectric permeability of the filling materials, such as air, is usually much less than that of MEE materials. In addition, electrode stratification or electrode–matrix debonding can often lead to the development of conductive cracks. When a conductive crack is loaded by an electrical and/or magnetic field parallel to the crack, electric charges in the conductive crack surfaces will rearrange themselves to develop an opposite field with the same magnitude, meaning that the electric field inside the conductive crack remains zero. Consequently, the charges in the upper and lower crack surfaces near the crack tip have the same sign. The phenomenon of these charges repelling each other has the effect of promoting crack propagation (Zhang et al., 2007). Therefore, the study of conductive cracks plays an important role in advancing our understanding of the failure behavior of MEE materials.

* Corresponding author. Tel.: +852 2859 2648; fax: +852 2559 5337.
E-mail addresses: klsu@hku.hk (R.K.L. Su), wjfeng9999@126.com (W.J. Feng).

http://dx.doi.org/10.1016/j.ijsolstr.2014.10.024
0020-7683/

Moreover, for some combinations of magnetoelectromechanical loads, a crack between dissimilar MEE materials can develop a crack face contact zone. The contact zone may exert cardinal influence on magnetoelectromechanical fields in the whole crack region, especially at the crack tips. In such cases, applying the classic Griffith crack model (without considering the contact zone) would lead to a physically unrealistic overlapping of the crack faces and an introduction of a false stress concentration near the crack tip. The aim of the contact zone is to eliminate these crack face overlapping zones and to determine the true open crack formed and the corresponding fracture parameters. Recently, Herrmann et al. (2010) and Ma et al. (2012) for the first time extended the contact zone model to interface crack problems of MEE materials, whereupon the oscillating singularity on the crack tip is eliminated.

However, to our best knowledge, an electrically conductive crack in an MEE bimaterial system has not been studied yet, despite the possibility of the appearance of large contact zones for such cracks under the action of electric and magnetic fields. This situation is significantly different from the aforementioned cases of the interface crack models proposed by Herrmann et al. (2010) and Ma et al. (2012), in which electrical and/or magnetic loads are perpendicular to the crack face and exert only a small influence on crack face contact. In the present study, we consider a new model, namely an electrically conductive crack with a frictionless contact zone in an MEE bimaterial system under the action of mechanical loading as well as the electrical and magnetic fields parallel to the crack faces. The expressions for contact zone length, stress intensity factors as well as electrical and magnetic field intensity factors are derived. Numerical results demonstrate that the contact zone indeed exists and can be found mathematically. Additionally, a significant influence of the electric field on the length of the contact zone and other fracture parameters is observed. These obtained results and/or conclusions could be of particular interest to the analysis and design of smart sensors/actuators composed of magnetoelectroelastic composite laminates.

2. Basic equations

The governing equations and general solutions for MEE half-spaces in a Cartesian coordinate system are consistent with those in Feng et al. (2011). Hence, for brevity, those equations will not be presented in this paper.

For the following analysis related to the conducting crack, it is convenient to introduce the vectors

$$\mathbf{C} = \{u_1', u_2', u_3', D_3, B_3\}^{\mathrm{T}}, \quad \mathbf{Y} = \{\sigma_{31}, \sigma_{32}, \sigma_{33}, E_1, H_1\}^{\mathrm{T}}, \tag{1}$$

where u_1, u_2 and u_3 are the mechanical displacement components; E_1 and H_1 are the components of the electrical field and the magnetic field respectively; σ_{31}, σ_{32} and σ_{33} are the components of the stress; D_3 and B_3 are the components of the electrical displacement and the magnetic induction respectively and the prime means the differentiation on x_1. Combined with the general solutions, these vectors can be written in the form (Loboda et al., 2014)

$$\mathbf{C} = \mathbf{M}\mathbf{f}'(z) + \overline{\mathbf{M}}\overline{\mathbf{f}}'(\bar{z}), \tag{2}$$

$$\mathbf{Y} = \mathbf{N}\mathbf{f}'(z) + \overline{\mathbf{N}}\overline{\mathbf{f}}'(\bar{z}), \tag{3}$$

where $\mathbf{f}(z) = \{\mathbf{f}_1(z_1), \mathbf{f}_2(z_2), \mathbf{f}_3(z_3), \mathbf{f}_4(z_4), \mathbf{f}_5(z_5)\}^T$, $z_j = x_1 + p_j x_3$ $(j = 1, 2, \ldots, 5)$ and the matrices $\mathbf{M}$ and $\mathbf{N}$ are found by means of the reconstruction of the matrices $\mathbf{A}$ and $\mathbf{B}$ in Feng et al. (2011). They can be expressed as

$$\mathbf{M} = \{a_{1j}, a_{2j}, a_{3j}, b_{4j}, b_{5j}\}^{\mathrm{T}},$$
$$\mathbf{N} = \{b_{1j}, b_{2j}, b_{3j}, -a_{4j}, -a_{5j}\}^{\mathrm{T}}, \quad j = 1, 2, \ldots, 5. \tag{4}$$

3. Statement of the problem

Fig. 1 shows an electrically conducting crack located at $c \leqslant x_1 \leqslant b$, $x_3 = 0$ between two semi-infinite MEE half-spaces $x_3 > 0$ and $x_3 < 0$ with material properties defined by the following material constants $c_{ijks}^{(1)}$, $e_{iks}^{(1)}$, $h_{iks}^{(1)}$, $d_{is}^{(1)}$, $\alpha_{is}^{(1)}$, $\mu_{si}^{(1)}$ and $c_{ijks}^{(2)}$, $e_{iks}^{(2)}$, $h_{iks}^{(2)}$, $d_{is}^{(2)}$, $\alpha_{is}^{(2)}$, $\mu_{si}^{(2)}$, respectively. The half-spaces are assumed to be loaded at infinity with uniform stresses $\sigma_{33}^{(m)} = \sigma_0$, $\sigma_{31}^{(m)} = \tau_0$, electrical field $E_1^{(m)} = E_0$ and magnetic field $H_1^{(m)} = H_0$ (where $m = 1$ stands for the upper domain and $m = 2$ for the lower domain).

Furthermore, the crack surfaces are assumed to be traction-free for $x_1 \in (c, a) = L_1$ whereas they are in frictionless contact for $x_1 \in (a, b) = L_2$, and the position of Point a is arbitrarily chosen for the time being. It has been shown that the longer contact zone develops at the right crack tip for the shear stress at infinity $\tau_0 > 0$ if the lower material is softer than the upper one and it develops for $\tau_0 < 0$ in the opposite case (Loboda, 1998; Herrmann et al., 2001). Also it is revealed by Dundurs and Gautesen (1988) and Kharun and Loboda (2003) that neglecting the left short contact zone, the oscillating singularity at the left crack tip will not significantly influence the stress and strain fields at the right crack tip. Therefore, in the present study only the contact zone at the right crack tip is considered. Certainly, a contact zone at the left crack-tip can be treated similarly.

Since the load and the displacement u_2 of the vector-function $(u_1, u_2, u_3, \varphi, \phi)$, in which φ and ϕ are the electrical potential and magnetic potential respectively, decouples in the (x_1, x_3)-plane from the components $(u_1, u_3, \varphi, \phi)$, in the following sections, our attention will be focused on the components $(u_1, u_3, \varphi, \phi)$ in a generalized plane strain condition.

Thus, for the present interface crack problem, the continuity and boundary conditions at the interface can be written in the following form:

$$[\tilde{\mathbf{C}}(x_1)] = \mathbf{0}, \quad [\tilde{\mathbf{Y}}(x_1)] = \mathbf{0}, \quad x_1 \notin (c, b), \tag{5a}$$

$$\sigma_{13}^{(m)}(x_1, 0) = 0, \quad \sigma_{33}^{(m)}(x_1, 0) = 0, \quad E_1^{(m)}(x_1, 0) = 0, \quad [H_1(x_1)] = 0,$$
$$[B_3(x_1)] = 0, \quad x_1 \in L_1, \tag{5b}$$

$$[u_3(x_1)] = 0, \quad \sigma_{13}^{(m)}(x_1, 0) = 0, \quad [\sigma_{33}(x_1)] = 0, \quad E_1^{(m)}(x_1, 0) = 0,$$
$$[H_1(x_1)] = 0, \quad [B_3(x_1)] = 0, \quad x_1 \in L_2, \tag{5c}$$

where

$$\begin{aligned}[\tilde{\mathbf{C}}(x_1)] &= \left\{[u_1'(x_1)], [u_3'(x_1)], [D_3(x_1)], [B_3(x_1)]\right\}^{\mathrm{T}} \\ &= \left\{u_1'(x_1, 0^+) - u_1'(x_1, 0^-), u_3'(x_1, 0^+) - u_3'(x_1, 0^-), D_3(x_1, 0^+)\right. \\ &\quad \left. - D_3(x_1, 0^-), B_3(x_1, 0^+) - B_3(x_1, 0^-)\right\}^{\mathrm{T}},\end{aligned} \tag{6a}$$

$$\begin{aligned}[\tilde{\mathbf{Y}}(x_1)] &= \{[\sigma_{31}(x_1)], [\sigma_{33}(x_1)], [E_1(x_1)], [H_1(x_1)]\}^{\mathrm{T}} \\ &= \left\{\sigma_{31}(x_1, 0^+) - \sigma_{31}(x_1, 0^-), \sigma_{33}(x_1, 0^+)\right. \\ &\quad \left. - \sigma_{33}(x_1, 0^-), E_1(x_1, 0^+) - E_1(x_1, 0^-), H_1(x_1, 0^+) - H_1(x_1, 0^-)\right\}^{\mathrm{T}},\end{aligned} \tag{6b}$$

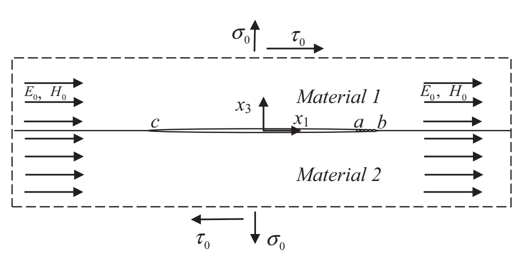

Fig. 1. An electrically conductive interface crack with frictionless contact zones under remote mixed mode mechanical load σ_0, τ_0, electrical field E_0 and magnetic field H_0.