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Free-edge interlaminar stress analysis of piezo-bonded composite laminates under symmetric electric excitation

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ABSTRACT

A stress function-based approach is proposed to analyze the free-edge interlaminar stresses of piezo-bonded symmetric laminates. The proposed method satisfies the traction free boundary conditions, as well as surface free conditions. The symmetric laminated structure was excited under electric fields that can generate induced strain, resulting in pure extension in the laminated plate. The governing equations were obtained by taking the principle of complementary virtual work. To verify the proposed method, cross-ply, angle-ply and quasi-isotropic laminates were analyzed. The stress concentrations predicted by the present method were compared with those analyzed by the finite element method. The results show that the stress function-based analysis of piezo-bonded laminated composite structures is an efficient and accurate method for the initial design stage of piezo-bonded composite structures.

1. Introduction

Recently, piezoelectric actuators and sensors are most commonly used in smart structures, modern control engineering and energy harvesting industries, because of their large electromechanical coupling effect, wide bandwidth and quick response (Kapuria et al., 2010; Kim et al., 2011). Fiber reinforced laminated composite materials have many advantages, compared with other metal and nonmetal materials in physics (Herakovich, 2012). These composite materials could be good candidates for enhancement, to replace the substrate materials of smart structures. The development of piezo-bonded composite laminates with their mathematical modeling provides many engineering applications (Chopra, 2002). These smart structures could be applied in many advanced engineering fields, such as aircraft structures, satellites, large space structures, and auto-motives. However, delamination is one of the major failure mechanisms in piezo-bonded composite laminates, due to interlaminar stresses at layer interfaces.

Since the last several decades, three-dimensional interlaminar stress and stress singularity study of these structures have received considerable attention (Pipes and Pagano, 1970; Rhee et al., 2006; Kim et al., 2008, 2010; Izadi and Tahani, 2010; Lee et al., 2011; Kassapoglou and Lagace, 1986). A large volume of literature achievements has been established from experimental, as well as theoretical investigations. The presence of material and geometric discontinuities resulted in stress concentrations at the free edges, which is also referred to as free edge effects or boundary layer effects (Wang and Choi, 1982a,b). Free-edge effects are critical to interlaminar failure or delamination of piezo and composite layers. Due to the piezoelectric coupling effects in piezo-bonded composite laminates, the free edge effects become more complicated. Analysis of these localized interlaminar stresses at the free edge is of great importance in the initial design of piezoelectric composite laminates.

There are various theories (Ghugal and Shimpi, 2002), based on displacement fields or stress fields, for predicting the flexural response of laminated plates with surface-bonded or embedded induced strain actuators. Among the displacement fields-based theories, classical lamination theory (CLT) (Lee, 1990; Konieczny and Woźniak, 1994; Wang et al., 1997), three equivalent single-layer shear deformation theories (ESLSDT) (Kabir, 1996; Reddy, 1999), layerwise shear deformation theory (LWSDT) Robbins and Reddy, 1993; Zhu and Lam, 1998; Kim et al., 2002a,b; Kim et al., 2006) and finite element methods (Chandrashekhara and Agarwal, 1993; Detwiler et al., 1995) are the most popularly used theories. However, CLT shows great inadequacy for the stress analysis of piezo-bonded composite laminates, as CLT assumes a linear displacement distribution across the thickness of entire laminates, and neglects the transverse shear deformation, which is necessary for moderately thick and thick laminates. ESLSDT cannot recover transverse shear stress continuity in the thickness direction by using the constitutive relations, which are discontinuous at interfaces between layers, and are against equilibrium conditions. LWSDT are the most accurate in displacement fields-based theories, but they are more computationally inefficient than

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ESLSDT, due to their demand for a large number of unknown variables.

Another approach to analyze laminated composite structure is stress function-based theory, which can fully satisfy not only stress continuity, but also traction free boundary conditions. These theories have great potential in the future study of interlaminar stress analysis of piezo-bonded composite laminates. After Spilker and Chou (1980) demonstrated the importance of satisfying the traction free boundary conditions at the free edges, Yin (1994a,b) proposed a variational method, using piecewise polynomial approximations based on stress-based layerwise theory (SBLWT). The stress functions involved in his approach not only satisfy pointwise equilibrium equation, but also continuity of interlaminar stress over each layer, and at the layer interfaces. Flanagan (1994) proposed a solution method based on a series expansion of mode shapes of a clamped–clamped beam, for determining the free-edge stresses in composite laminates. His approach can be concluded in stress-based equivalent single-layer theory (SBESLT) that is computationally more efficient than Yin's work. While Flanagan's method can well predict the interlaminar stresses along the in-plane direction, the interlaminar stresses along the thickness direction show oscillations. Flanagan's work was improved by Cho and Kim (2000) and Kim et al. (2000), by using an extended Kantorovich method. In their works, converged stress distributions obtained under extension, bending, twisting and thermal loading are independent of the number of initially assumed stress functions, and the oscillations appearing in the thickness direction can be reduced, or even eliminated.

Actuating force of piezoelectric layer causes stress concentration at the free edge of smart composite laminates. The concentrated stress could initiate debonding of the actuator and lead failure of the smart composite laminates. However, most researches have been focused on global responses of the smart composite laminates such as vibration control, energy harvesting and structural health monitoring and so on (Chopra, 2002; Kim et al., 2011). Kapuria and Kumari focused on stress concentration of piezolaminated panels under electromechanical coupling load (Kapuria and Kumari, 2013). They provided accurate three dimensional piezoelasticity solutions using extended Reissner-type variational principle and extended Kantorovich method. In this work, we will investigate stress concentration of piezo-bonded smart composite laminates under piezoelectric excitation using stress variables only. The principle of complementary virtual work is adopted to derive governing equations. Various layup configurations are studied, to verify the proposed approach. The proposed method is also evaluated by comparing finite element analysis results, using the commercially available package ANSYS.

2. Formulations

Fig 1 shows a symmetric configuration of piezo-bonded composite laminates with free edges. Two piezoelectric actuators are bonded on the top and bottom surfaces of the laminated composites. The composite laminates consist of orthotropic materials, and have arbitrary fiber angles with respect to the x axis. The thicknesses of piezoelectric actuators and composite laminates are considered the same in each layer, for convenience. This structure can be extended, bended, and twisted under electric excitation, due to the electro-mechanical coupling of the piezoelectric actuators.

Based on the linear elasticity, the general form of the constitutive equations can be expressed for each layer in Eq. (1). Induced strains ($[d][E]$) only exist in piezoelectric layers. A piezoelectric strain matrix $[d]$ is defined at each layer, and it has zero values for composite layers. Pure extension by the piezoelectric excitation is considered in the present study. Therefore, electric fields $\{E\}$ are applied through the thickness direction only, and E_1 and E_2 are zero.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & 0 \\ S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \tag{1}$$

The first row of Eq. (1) can be rewritten into the following form.

$$\sigma_1 = \frac{\varepsilon_1 - S_{1j}\sigma_j - d_{31}E_3}{S_{11}}, \quad (j = 2, 3, \ldots, 6) \tag{2}$$

Substituting Eq. (2) into Eq. (1), all other strains can then be expressed as follows.

$$\varepsilon_i = \hat{S}_{ij}\sigma_j + \frac{S_{i1}}{S_{11}}\varepsilon_1 + \hat{d}_{3i}E_3, \quad (i,\ j = 2, 3, \ldots, 6) \tag{3}$$

where,

$$\hat{S}_{ij} = S_{ij} - \frac{S_{i1}S_{1j}}{S_{11}}, \quad \hat{d}_{3i} = d_{3i} - \frac{S_{i1}}{S_{11}}d_{31} \tag{4}$$

The boundary conditions for the given geometric configuration at the free edges, and at the top and bottom surfaces, are given in the following equations.

$$\begin{aligned} &\sigma_2 = \sigma_4 = \sigma_6 = 0 \quad \text{at } y = 0, b \\ &\sigma_3 = \sigma_4 = \sigma_5 = 0 \quad \text{at } z = \pm \mathrm{H}/2 \end{aligned} \tag{5}$$

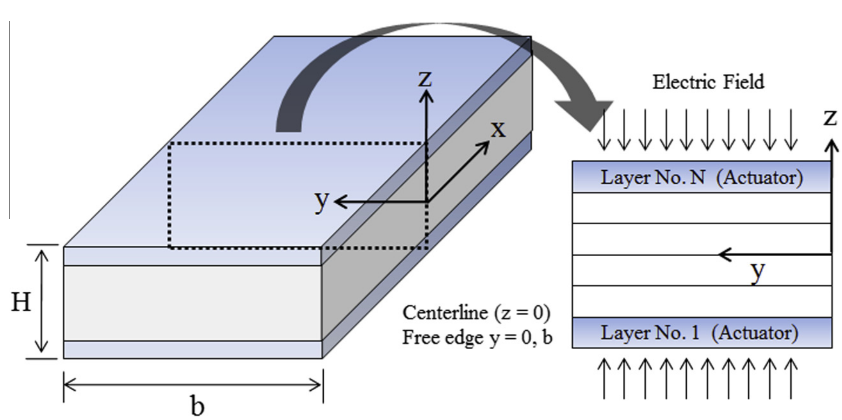

Fig. 1. Geometry of piezo-bonded composite laminate.