International Journal of Solids and Structures 47 (2010) 2017–2033

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Nonlinear finite element analysis of inflatable beams made from orthotropic woven fabric

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A R T I C L E I N F O

Article history:
Received 3 November 2009
Received in revised form 26 February 2010
Available online 3 April 2010

Keywords:
3D inflatable beams
2D orthotropic woven fabric
Nonlinear finite element

A B S T R A C T

This paper was devoted to the three-dimensional nonlinear finite element analysis of inflatable beams. The beams under consideration are made of modern textile materials and can be used as a load-bearing beams or arches when inflated. A 3D Timoshenko beam with a homogeneous orthotropic woven fabric (OWF) was proposed. The model took into account the geometric nonlinearities and the follower force resulting from the inflation pressure. The use was made of the usual total Lagrangian form of the virtual work principle to perform the nonlinear equilibrium equations which were discretized by the finite element method. Two kinds of solutions were then investigated: finite elements solutions for linearized problems which were obtained by the means of the linearization around the prestressed reference configuration of the nonlinear equations and nonlinear finite element solutions which were performed by the use of an optimization algorithm based on the Quasi-Newton method. As an example, the bending problem of a cantilever inflated beam under concentrated load was considered and the deflection results improve the existing theoretical models. As these beams are made from fabric, the beam models were validated through their comparison with a 3D thin-shell finite element model. The influence of the material effective properties and the inflation pressure on the beam response was also investigated through a parametric study. The finite elements solutions for linearized problems were found to be close to the theoretical results existing in the literature. On the other hand, the results for the nonlinear finite element model were shown to be close to the results for the linearized finite elements model in the case of high mechanical properties and the nonlinear finite element model was used to improve the linearized model when the mechanical properties of the fabric are low.

1. Introduction

In the recent decades, structural applications with inflatable beams or arches with modern textile materials have been growing, requiring great effort on the development of analysis. The advantage of using modern textile materials for these beams and arches over conventional materials is that the former can be tailored to specific requirements of certain applications, easy to deploy, lightweight and have a low storage volume.

Nowadays, inflatable beams pose significant challenges to the analysts. In the numerical modelling of inflatable beams, significant prior research have been conducted.

Steeves (1975, 1978) has investigated the load–deflection behaviour of pressurized beams based on linearly elastic theory, and has developed a linear pressurized fabric beam element that included a pressure stiffening term. Quigley et al. (2003) and Cavallaro et al. (2003) have used this finite element to predict the linear load–deformation response of inflated fabric beams. However, the pressure stiffening term in Steeves's element treated the axial pressure resultant as an externally applied, stiffening tension force. This formulation predicted an unbounded increase in beam stiffness with increasing inflation pressure. Wielgosz and Thomas (2002, 2003) and Thomas and Wielgosz (2004) have studied the load–deflection behaviour of highly inflated fabric tubes and panels, and have developed a specialized beam finite element using Timoshenko beam theory. In their approach, the force generated by the internal pressure has been treated as a follower force which has accounted for pressure stiffening effects. Bouzidi et al. (2003) have presented theoretical and numerical developments of finite elements for axisymmetric and cylindrical bending problems of pressurized isotropic membranes. The external loading has been mainly a normal pressure to the membrane and the developments have been made under the assumptions of follower forces, large displacements and finite strains. The total potential energy has been minimized, and the numerical solution has been obtained by using an optimization algorithm. Suhey et al. (2005) have presented a numerical simulation and design of an inflatable

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doi:10.1016/j.ijsolstr.2010.03.030

Nomenclature

- *Coordinates systems*

$(\underline{l}, \underline{t}, \underline{n})$ warp, weft, normal directions of the fabric
$(X,\ Y,\ Z)$ cartesian coordinates
ξ reference coordinate
$(\underline{e}_X, \underline{e}_Y, \underline{e}_Z)$ unit vectors of the cartesian coordinates
$\varphi = (\underline{e}_Z, \underline{n})$ angle

- *Mechanical properties*

E_l modulus of elasticity in $\underline{l}$ direction of the orthotropic fabric
E_t modulus of elasticity in $\underline{t}$ direction of the orthotropic fabric
G_{lt} in-plane shear modulus of the orthotropic fabric
ν_{lt} Poisson's ratio due to the loading in the $\underline{l}$ direction and contraction in the $\underline{t}$ direction
ν_{tl} Poisson's ratio due to the loading in the $\underline{t}$ direction and contraction in the $\underline{l}$ direction

- *Internal forces*

N axial force
T_y, T_z shear force along y and z axes
M_y, M_z moments around y and z axes

- *Beam geometry*

l_ϕ natural length of the inflatable beam
R_ϕ natural radius of the inflatable beam
t_ϕ natural thickness of the inflatable beam
l_o reference length of the inflatable beam
R_o reference radius of the inflatable beam
t_o reference thickness of the inflatable beam
A_o reference cross-section area of the inflatable beam
I_o reference moment of inertia of the inflatable beam

- *Loads*

F intensity of the service load
f normalized load
F_X, F_Y, F_Z components of concentrated loads
f_x, f_y, f_z components of the distributed load
M_Y, M_Z components of bending moments
F_w wrinkling load

- *Pressure, pressure forces*

p inflation pressure
p_n normalized pressure
$F_p = p\pi R_o^2$ pressure force
N^o axial force due to the inflation pressure

- *Kinematics*

$\underline{U}$ displacements field
u axial displacement
v, w deflections along Y and Z axes
θ_Y, θ_Z rotations around Y and Z axes

- *Tensors*

$\underline{\underline{E}}$ Green–Lagrange tensor
$\underline{\underline{S}}$ second Piola–Kirchhoff tensor
$\underline{\underline{R}}$ rotation matrix

- *Functions*

δW_{ext}^d external virtual work of the service load
δW_{ext}^p external virtual work of the pressure load

open-ocean-aquaculture cage using nonlinear finite element analysis of isotropic membrane structures. Numerical instability caused by the tension-only membrane has been removed by adding an artificial shell with small stiffness. The finite element model has been compared with a modified beam theory for the inflatable structure. A good agreement has been observed between the numerical and theoretical results. Le Van and Wielgosz (2005) have introduced finite rotations and an energy approach which has built on the earlier work of Fichter (1966) to analytically study the bending and buckling of highly inflated isotropic fabric beams. Le Van and Wielgosz (2007) have discretized the nonlinear equations obtained in Le Van and Wielgosz (2005) to carry out a finite element formulation for linearized problems of highly inflated isotropic fabric beams. Their numerical results obtained with the beam element have been shown to be close to their 3D isotropic fabric membrane finite element and analytical results obtained in Le Van and Wielgosz (2005). Davids (2007) and Davids and Zhang (2008) have derived a Timoshenko beam finite element for nonlinear load–deflection analysis of pressurized isotropic fabric beams and the numerical examination of the effect of pressure on the beam load–deflection behaviour. The basis of their element formulation has been an incremental virtual work expression that has included explicitly the work done by the pressure. Parametric studies have been also investigated to demonstrate the importance of including the work done by the pressure in their models. More recently, Malm et al. (2009) have used 3D isotropic fabric membrane finite element model to predict the beam load–deformation response. Comparison between the finite element model-predicted load–deflection response and beam theory has been shown the accuracy of the conventional beam theory to load–deformation for the isotropic fabric airbeam. In these former works, the fabric used to manufacture the beams was always supposed to be isotropic whereas this character is not the best one for this material.

Few papers deal with the case of the inflatable beams made of orthotropic fabric. Plaut et al. (2000) have studied the effect of the snow and wind loads on an inflated arch in the assumption of linear thin-shell theory of Sanders. They have used this theory to formulate the governing equations, which include the effect of the initial membrane stresses. The material was assumed to have a linearly elastic, nonhomogeneous and orthotropic behaviour. Approximate solutions have been obtained using the Rayleigh–Ritz method. Any study of the influence of the fabric orthotropic character has not been conducted. More recently, Apedo et al. (2009) have used the earlier work of Le Van and Wielgosz (2005) to analytically study the bending and the wrinkling problems of inflatable beams made of 2D OWFs. They have shown the importance of taken into account the orthotropic character of the fabric even if the effective properties in the two principal directions (warp and weft) are close.

Consequently, there is a need to develop efficient numerical techniques for predicting the nonlinear load–displacement response of inflatable beams and arches made of a 2D OWF. In this paper, this need is addressed through the development of a 3D Timoshenko beam element for the nonlinear analysis of inflatable 2D OWF beams. The present paper extends the work done in Apedo et al. (2009). The nonlinear equilibrium equations obtained by the virtual work principle are discretized by the finite element method. Two kinds of solutions are then investigated: finite elements solutions for linearized problems and nonlinear finite element solutions. As an example, the bending problem of a cantilever inflated beam under concentrated load is considered. To show the influence of the material properties on the beam response,