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Energy-based analytical model to predict the elastic critical behaviour of curved panels

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ABSTRACT

This paper presents an analytical study on the elastic critical behaviour of cylindrically curved panels under pure compressive stresses, for which an energy formulation is developed. Firstly, this energy formulation is described, the general assumptions are stated, the degrees-of-freedom and the displacement functions are defined and, using strain-displacement relations, the strain energy and the potential energy are derived. Secondly, the resulting general energy formulation is used to obtain, whenever feasible, simple expressions or, otherwise, values of the elastic critical stress of simply supported cylindrically curved panels under pure compression. A discussion on the number of degrees-of-freedom necessary to obtain accurate results is also presented. The analytical results are compared to numerical (finite element) results obtained by the authors. Finally, a parametric study is made regarding the influence of constraining (or not) the panel longitudinal edges on the elastic critical stress.

1. Introduction

A thin shell is a thin-walled three-dimensional body for which one dimension (i.e. the thickness) is significantly lower than the other two dimensions and it is characterised by its non-plane initial shape (i.e. finite radius of curvature). In other words, shells have generally more intricate behaviour than plates due to their initial curvature. Depending on the type of variation of curvature of the shell middle surface, it can be classified as cylindrical, conical, spherical, ellipsoidal, toroidal and torispherical [1] and, these different shapes, together with the support conditions, loading type and constitutive laws of the material, will determine the shell behaviour. One of the most interesting shell structures is the cylindrical panel, which displays constant finite curvature along its transversal direction and null curvature along its longitudinal direction, does not exhibit full geometric revolution and it is supported along the four edges (two longitudinal and two transversal). In this paper, we consider cylindrically curved panels as thin shallow shells, similar to slightly curved plates [2]. It is possible to define a thin shallow shell on the basis of the following conditions [3]:

$$\left(\frac{\partial Z(x,y)}{\partial x}\right)^2 < 0.05 \qquad \left(\frac{\partial Z(x,y)}{\partial y}\right)^2 < 0.05 \tag{1}$$

where $Z(x,y)$ is the function representing the position of the shell middle surface. According to Koiter [4] a shell is said to be shallow if the relation between the characteristic wavelength of its deformed configuration l, and the smallest radius of curvature of the middle surface R_{min}, is negligible, i.e. $l/R_{min} \ll 1$. Vlasov [5] defined shallow shell as a shell whose rise is limited to 20% of the smallest dimension of the shell in its plane (projection on the coordinate plane Oxy). Later in 1959, Novozhilov [6] showed that this definition leads to errors exceeding 5%.

The most successful theory to study thin shallow shells is that proposed by Donnell-Mushtari-Vlasov [4,7,8] nonlinear theory, or simply DMV nonlinear theory. In order to introduce DMV theory, a brief description of the shell geometry is made. Using Fig. 1 as reference it is possible to define infinitesimal distances ds_x and ds_y. These distances can be given by the following expressions:

$$ds_x = Adx \quad ; ds_y = Bdy \tag{2}$$

where A and B are the Lamé coefficients given by the following expressions [7].

$$A = \left[\left(\frac{\partial X}{\partial x}\right)^2 + \left(\frac{\partial Y}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial x}\right)^2\right]^{1/2} \qquad B = \left[\left(\frac{\partial X}{\partial y}\right)^2 + \left(\frac{\partial Y}{\partial y}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]^{1/2} \tag{3}$$

where $X = X(x,y)$, $Y = Y(x,y)$ and $Z = Z(x,y)$.

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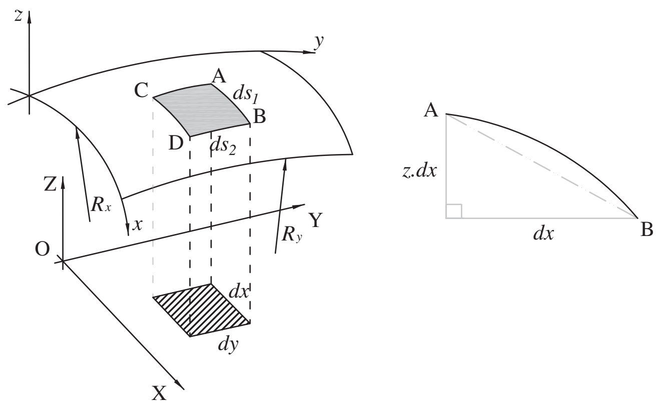

Fig. 1. Geometric definition of the relationships in expression (2).

DMV nonlinear theory assumes the fundamental hypotheses of the classical theory formulated by Love [9]. Besides these hypotheses, DMV nonlinear theory also assumes that the shell shows infinitesimal deformations and moderate rotations, being suitable to analyse shallow shells. DMV theory takes into account rotations and the kinematic relationships incorporate them as follows:

$$
\begin{aligned}
\varepsilon_{x,0} &= \frac{1}{A}\frac{\partial u}{\partial x} + \frac{1}{AB}\frac{v\partial A}{\partial y} + \frac{w}{R_x} + \frac{1}{2}\left(-\frac{1}{A}\frac{\partial w}{\partial x}\right)^2 \\
\varepsilon_{y,0} &= \frac{1}{B}\frac{\partial v}{\partial y} + \frac{1}{AB}\frac{u\partial B}{\partial x} + \frac{w}{R_y} + \frac{1}{2}\left(-\frac{1}{B}\frac{\partial w}{\partial y}\right)^2 \\
\gamma_{xy,0} &= \frac{1}{A}\frac{\partial v}{\partial x} + \frac{1}{B}\frac{\partial u}{\partial y} - \frac{1}{AB}\left(\frac{v\partial B}{\partial x} + \frac{v\partial A}{\partial y}\right) + \left(-\frac{1}{A}\frac{\partial w}{\partial x}\right)\left(-\frac{1}{B}\frac{\partial w}{\partial y}\right) \\
\chi_x &= -\frac{1}{A^2}\frac{\partial^2 w}{\partial x^2} + \frac{1}{A^3}\frac{\partial A}{\partial x}\frac{\partial w}{\partial x} - \frac{1}{AB^2}\frac{\partial A}{\partial y}\frac{\partial w}{\partial y} \\
\chi_y &= -\frac{1}{B^2}\frac{\partial^2 w}{\partial y^2} + \frac{1}{B^3}\frac{\partial B}{\partial y}\frac{\partial w}{\partial y} - \frac{1}{A^2 B}\frac{\partial B}{\partial x}\frac{\partial w}{\partial x} \\
\chi_{xy} &= -\frac{1}{AB}\frac{\partial^2 w}{\partial x \partial y} + \frac{1}{A^2 B}\frac{\partial A}{\partial y}\frac{\partial w}{\partial x} - \frac{1}{AB^2}\frac{\partial B}{\partial x}\frac{\partial w}{\partial y}
\end{aligned} \tag{4}
$$

Where, u, v and w are functions describing the displacement field in longitudinal, transverse and out-of-plane direction, respectively, $\varepsilon_{x,0}$ and $\varepsilon_{y,0}$ are the membrane strain at x- and y-direction, $\gamma_{xy,0}$ is the membrane distortion between x- and y-direction, χ_x and χ_y are the changes in curvature in x- and y-direction and χ_{xy} is the twist in curvature between in x- and y-direction.

Finally, it can be seen that the Lamé parameters can be specified to obtain kinematic relationships for specific geometries, such as the rectangular flat plate ($A=1, B=1, R_x=\infty, R_y=\infty$) and circular cylindrical shell ($A=1, B=R_y, R_x=\infty$). For the case of nonlinear shallow shells, the Lamé parameters are $A=1$, $B=1$ and both R_x and R_y are finite. Therefore, it is acceptable, when analysing shallow shells, to consider the curvilinear (orthogonal) system of coordinates. In conclusion, the intrinsic geometry of a shallow shell is identical to the geometry of a plane of its projection. This actually represents the first basic assumption of the theory of shallow shells [3]. The other assumptions of shallow shells theory are that (i) the effect of transverse shear forces in in-plane equilibrium equations is negligible and (ii) the influence of the deflections w in the bending response of the shell predominates over the influence of the in-plane displacements u and v.

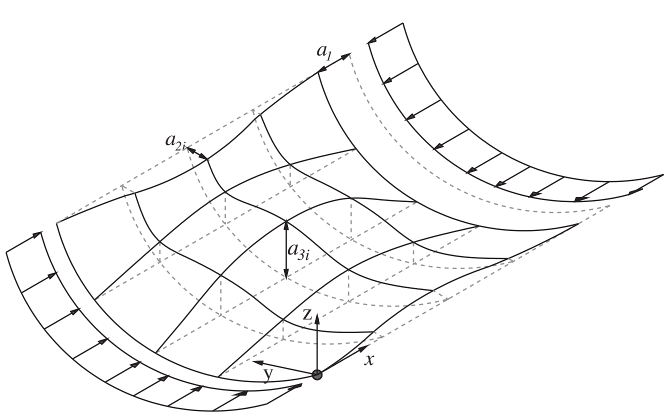

Fig. 2. Degrees-of-freedom considered in the analysis.

The elastic buckling of cylindrically curved panels under pure compression was studied by Redshaw [10], Timoshenko [11], Stowell [12] and Batdorf [13], who proposed analytical expressions for the calculation of critical stresses. Despite being a classical theme, the study buckling behaviour of curved panels is recently gaining importance since this type of structural elements is being employed in the design of slender webs in steel bridge girders to resist shear and flexural compression [14], as an example. Additionally, there are some accepted and commonly used methods to compute the elastic critical stress of cylindrically curved panels but, as some researchers have been claiming in their research outcomes, they are outdated [15,16]. Because of these reasons, recent investigations by Domb & Leigh [15] and Martins et al. [17], were carried out. They performed extensive numerical studies on the buckling behaviour of shallow cylindrical panels and proposed methodologies to calculate accurately their elastic critical stress. The proposed methodology by Martins et al. [17] although quite accurate (Stowell proposal presents errors up to 20%) lacks a mechanical background and it is purely calibrated with numerical results. For that reason, one of the objectives of this paper is to derive simple to use analytical closed-form expressions maintaining a strong mechanical background.

2. Energy formulation

2.1. General

The semi-analytical study presented herein is based on previous studies by Thompson & Hunt [18] and Simões da Silva [19]. The approach followed by Thompson & Hunt ignores the transverse and shear membrane stresses. From a physical point of view, this assumption means that the panel is seen as a group with infinite number of infinitely thin strips unable to transfer shear and/or membrane stresses to each other, but acting together during the deformation imposed by the external applied loads. Although this simplistic discretization assumption, this energy formulation is still able to incorporate the main features of the plate's buckling and postbuckling behaviour [18]. However, in this paper, this simplification is dropped making the panel able to transfer shear and membrane stresses in both directions. It should be mentioned that while for flat panels accounting for nonlinear terms in the y-direction (quadratic terms in second expression of Eqs. (4) and (10)) does not play any role in obtaining the elastic critical stress, the same is not true for curved panels, where the consideration for these extra nonlinear terms is crucial to obtain accurate results for both the elastic critical stress and the postbuckling path.

2.2. Strain energy and potential of external loads

For a general shell element, the total potential energy of an elastic body depends on the membrane strain energy U_m (Eq. (5)), bending strain energy U_b (Eq. (5)), and potential of external loads (Eq. (5)) [7]