Journal of Rock Mechanics and Geotechnical Engineering 7 (2015) 434–439

Contents lists available at ScienceDirect

Journal of Rock Mechanics and Geotechnical Engineering

journal homepage: www.rockgeotech.org

Full length article

Strength criterion for rocks under compressive-tensile stresses and its application

Mingqing You*

School of Energy Science and Engineering, Henan Polytechnic University, Jiaozuo, 454010, China

ARTICLE INFO

Article history:
Received 17 January 2015
Received in revised form
16 May 2015
Accepted 20 May 2015
Available online 16 June 2015

Keywords:
Strength criteria
Tensile strength
Exponential criterion
Hydraulic fracturing

ABSTRACT

Estimating in-situ stress with hydraulic borehole fracturing involves tensile strength of rock. Several strength criteria with three parameters result in tensile strengths with great differences, although they may describe the relation between strength of rock and confining pressure with low misfits. The exponential criterion provides acceptable magnitudes of tensile strengths for granites and over-estimates that for other rocks, but the criterion with tension cut-off is applicable to all rocks. The breakdown pressure will be lower than the shut-in pressure during hydraulic borehole fracturing, when the maximum horizontal principal stress is 2 times larger than the minor one; and it is not the peak value in the first cycle, but the point where the slope of pressure-time curve begins to decline.

1. Introduction

Numerous tests have been carried out to determine the strengths of rocks under confining pressure (CP), as rocks in-situ are usually under compression state. However, tension usually appears in the vicinity of excavation and borehole, and the tensile strength of each rock is much lower than the compressive strength.

The direct tension test is difficult to perform for rock (You et al., 2006). In other hand, the Brazilian splitting test with rock disc is easy to carry out in laboratory and provides a reasonable estimation for the uniaxial tensile strength (UTS), although there are many issues argued all along (Fairhurst, 1964; Hudson et al., 1972; Efimov, 2009; Yu et al., 2009; You et al., 2011).

Many strength criteria have been proposed to describe the state of stresses in rock at failure, as reviewed in Yu (2002) and You (2011). Clearly, an ideal strength criterion needs to closely fit test data with acceptable accuracy over the stress state expected in practice. Therefore, the tensile strength predicted by a strength criterion is usually used in evaluating the criterion (Ghazvinian et al., 2008; Bineshian et al., 2012). Tensile strengths predicted by both the Coulomb criterion and the Griffith criterion are much higher than the measured magnitudes of almost all rocks, although the two criteria have clear physical backgrounds (Jaeger et al., 2007).

Another issue is the effect of compressive stress on tensile strength, i.e. strength criterion in tension-compression region. It has practical utilization in the in-situ stress estimation with hydraulic breakout of borehole and some cases of wellbore stability.

This paper discusses four criteria with three material-dependent parameters using test data of nine rocks from the published literature. The exponential criterion with tension cut-off is recommended and adopted to estimate in-situ stress with hydraulic borehole fracturing.

2. Strength criteria

Coulomb criterion was initially proposed in 1773 for determination of the shear strength of soil, and introduced for rocks later. It is a linear equation with the principal stresses as

$$\frac{\sigma_S}{\sigma_C} = 1 + m\frac{\sigma_3}{\sigma_C} \tag{1}$$

where σ_S is the major principal stress or rock compressive strength, σ_3 is the minor principal stress, σ_C is the uniaxial compressive strength (UCS), and m is a material-dependent parameter. However, test results from cylindrical specimens of rocks compressed under CP of $\sigma_2 = \sigma_3$ exhibit convex curves of strength. Therefore, many nonlinear criteria were proposed as modifications to the Coulomb criterion, and briefly reviewed as follows.

Hobbs (1964) proposed an empirical criterion with three parameters:

$$\frac{\sigma_S - \sigma_3}{\sigma_C} = 1 + m\left(\frac{\sigma_3}{\sigma_C}\right)^n \tag{2}$$

* Tel.: +86 391 3988278.
E-mail addresses: youmq@hpu.edu.cn, youmq640930@aliyun.com.

Peer review under responsibility of Institute of Rock and Soil Mechanics, Chinese Academy of Sciences.
1674-7755 © 2015 Institute of Rock and Soil Mechanics, Chinese Academy of Sciences. Production and hosting by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jrmge.2015.05.002

Its special case at $m = 2$ and $n = 1/2$ is the normal parabolic criterion (You, 2011):

$$\sqrt{\sigma_S} = \sqrt{\sigma_3} + \sqrt{\sigma_C} \quad (3)$$

The criterion with one parameter merely fits strengths of granular rocks better than the Coulomb criterion, nearly the same as the Hoek-Brown (H–B) criterion with two parameters.

The Murrell criterion (Murrell, 1965) was widely used at $n = 0.5$ (Mogi, 2007):

$$\frac{\sigma_S}{\sigma_C} = 1 + m\left(\frac{\sigma_3}{\sigma_C}\right)^n \quad (4)$$

The two criteria, Eqs. (2) and (4), are not applicable to negative σ_3 for power number n is less than 1, and are certainly beyond the consideration for compressive-tensile strength.

The Sheorey criterion (Sheorey et al., 1989) normalized with UCS is given in the following form:

$$\frac{\sigma_S}{\sigma_C} = \left(1 + m\frac{\sigma_3}{\sigma_C}\right)^n \quad (5)$$

The criterion proposed in Carter et al. (1991) is in the similar form. The derivative of σ_S to σ_3 for Eq. (5), and Eq. (4) as well, will be less than 1 when σ_3 is large enough. That means the differential stress $\sigma_S - \sigma_3$ will decrease with increasing CP. The phenomenon appears really for Solnhofen limestone (Mogi, 2007), and Indiana limestone (Schwartz, 1964) as well, within the test range of CP, as illustrated by You (2011). It is totally different from the common knowledge.

The most famous criterion in power form is the generalized H–B criterion (Hoek et al., 1992):

$$\frac{\sigma_S - \sigma_3}{\sigma_C} = \left(1 + m\frac{\sigma_3}{\sigma_C}\right)^n \quad (6)$$

The specific form at $n = 1/2$ is called the H–B criterion (Hoek and Brown, 1980) that has been widely used in rock engineering (Eberhardt, 2012). The criterion fails to describe strength of ductile rocks, such as limestone and marble under high CP.

Cohesion and friction in rocks do not act simultaneously at one point, and the cohesion will be replaced by the frictional resistance when crack initiates in the rock under compression (You, 2005a). The intact rock under shearing will yield and lose its cohesion, but cracks do not slide macroscopically to increase the friction to the maximum when CP is high enough. The differential stress $\sigma_S - \sigma_3$, or the maximum shear stress equivalently, has an upper limitation in rocks, and is able to be described with a general criterion (You, 2012):

$$\sigma_S - \sigma_3 = Q_\infty - (Q_\infty - Q_0)f(x) \quad (7)$$

where Q_0 is the UCS; Q_∞ is the limitation of differential stress when CP increases up to infinite; $f(x)$ is a monotonically decreasing function, and satisfies $f(0) = 1$, $f(\infty) = 0$, and $f'(0) = -1$; and x can be written as

$$x = \frac{(K_0 - 1)\sigma_3}{Q_\infty - Q_0} \quad (8)$$

where K_0 is the increasing rate of strength at $\sigma_3 = 0$.

The exponential criterion (You, 2009, 2010a) is a specific case of Eq. (7) at

$$f(x) = \exp(-x) \quad (9)$$

The fractional form

$$f(x) = 1/(1 + x) \quad (10)$$

for Eq. (7) is equivalent to the criterion in Rafiai (2011) and Bineshian et al. (2012), and the later manifested that the criterion was originally proposed in Bineshian (2000). In this paper, we called it as the fractional criterion, which is just parallel to the exponential criterion mentioned above.

The average principal stress $\sigma_m = (\sigma_1+\sigma_3)/2$ and the maximum shear stress $\tau_m = (\sigma_1-\sigma_3)/2$ are usually used to construct implicit strength criteria. In fact, the abscissa and ordinate will become $2\sigma_m$ and $\sqrt{2}\tau_m$, respectively, after the coordinates of the principal stresses with the same scale are rotated 45° counterclockwise. Therefore, the implicit criteria are not discussed here.

3. Fitting solutions of strength criteria and tensile strengths predicted

In strength criteria, there are always material-dependent parameters, which are determined by fitting the criteria to test data. Test data of nine rocks, presented in Table 1, including granite, limestone, marble, sandstone, and halite, are cited from the literature (Von Kármán, 1911; Schwartz, 1964; Carter et al., 1991; Haimson and Chang, 2000; Sriapai, 2010; You, 2010a) to evaluate the strength criteria. Average magnitude of strengths with the same CP is used as one datum.

Different solutions of fitting the criteria to test data will be obtained using various statistical methods. The least square method is mostly used for the convenience in mathematical calculation, but the fitting solution will depart significantly from normal data to reduce the squares deviation of abnormal data with huge error. Linear regression for a transformed equation of the H–B criterion may result in an imaginary number of UCS (You, 2010a, 2012). Therefore, the fitting solution on the least absolute deviation, i.e. the least mean misfit, is chosen in this paper.

The average values of the mean misfits for nine rocks are 2.9 MPa, 2.9 MPa, 3.1 MPa, and 3.5 MPa using the Sheorey criterion, the fractional criterion, the exponential criterion, and the generalized H–B criterion, respectively. Each criterion provides the least mean misfits for some rocks. The exponential criterion is the best one for three rocks.

Certainly, the misfit is not the single standard to evaluate strength criteria. As illustrated in You (2010a, 2012), the exponential criterion may expose a few abnormal data of Mizuho trachyte and Jinping sandstone with huge misfit. A new example of Maha Sarakham halite (Sriapai, 2010) is shown in Figs. 1 and 2. Clearly, the misfit of the exponential criterion is mainly pronounced for two data indicated with *A* and *B*, as shown in Fig. 1. At least, datum *A* may be pointed as an abnormal one. If the datum is deleted, then

Table 1
Test data of conventional triaxial compression of nine rocks.

Rock type	Number of test data	CP (MPa)	UCS (MPa)	Reference
Westerly granite	7	100	201	Haimson and Chang (2000)
Bonnet granite	13	40	226	Carter et al. (1991)
Tyndall limestone	9	40	52	Carter et al. (1991)
Indiana limestone	11	69	45	Schwartz (1964)
Carrara marble	6	162	137	Von Kármán (1911)
Georgia marble	10	69	30.6	Schwartz (1964)
Pottsville sandstone	10	62	62	Schwartz (1964)
Zhaogu sandstone	10	45	132.4	You (2010a)
Maha Sarakham halite	9	28	23	Sriapai (2010)