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On enhancement of vibration-based energy harvesting by a random parametric excitation

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ABSTRACT

An electromechanical linear oscillator with a random ambient excitation and telegraphic noise parametric excitation is considered as an energy harvester model. It is shown that a parametric colored excitation can have a dramatic effect on the enhancement of the energy harvesting. A close relation with mean-square stability of the oscillator is established. Four sources of the ambient excitation are considered: the white noise, the Ornstein–Uhlenbeck noise, the harmonic noise and the periodic function. Analytical expressions for stationary electrical net mean power are presented for all the considered cases, confirming the proposed approach.

1. Introduction

Vibration-based energy harvesting has recently received a great attention (see e.g. contributions to the book [1] and references therein). The aim of the research is to design a device that will harvest as much energy as possible, subject to constraints on the size, weight, and cost of the system. The main method of performing power harvesting is to use piezoelectric materials that can convert the ambient energy surrounding the device into electrical energy (see e.g. the review [2]). Majority vibration-based energy harvesters are designed as linear resonators to achieve the optimal performance by matching their natural frequencies with the ambient harmonic excitation frequencies. However, a slight shift of the excitation frequency will cause a dramatic reduction in the device's performance. Unfortunately, in the vast majority of practical cases, the ambient excitations are random with energy distributed over a wide frequency spectrum. To overcome this difficulty new approaches based on using important features of nonlinear vibratory systems have been intensively studied (see recent reviews [3–5]).

For the sake of demonstration a key idea of the method considers a simple vibratory system under a random external excitation. It is of course the linear harmonic oscillator

$$\ddot{y}+\gamma\dot{y}+\omega^2 y=\eta(t), \tag{1}$$

where $\gamma>0$ is a damping parameter, ω is the natural frequency, and $\eta(t)$ is a Gaussian white noise with intensity D,

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$\langle\eta(t)\eta(s)\rangle = 2D\delta(t-s)$. It is easy to show that the second-order moments

$$z_1 = E[y^2], \quad z_2 = E[\dot{y}^2], \quad z_3 = E[y\dot{y}]$$

satisfy the following set of equations [6]:

$$\dot{z}_1 = 2z_3, \quad \dot{z}_2 = -2\gamma z_2 - 2\omega^2 z_3 + 4D, \; \dot{z}_3 = z_2 - \gamma z_3 - \omega^2 z_1. \tag{2}$$

One can get readily from (2) the stationary moments ($t\to\infty$). Then for the stationary mean kinetic energy of the oscillator $E_0 = z_{2,(\mathrm{st})}/2$ we have the expression

$$E_0 = \frac{D}{\gamma}. \tag{3}$$

It is worth mentioning that it has been recently demonstrated in Ref. [7] that a linear as well as nonlinear system (with the exception of multistable systems) under an external white noise excitation can absorb on average a limited amount of power, which is proportional to the mass of the system and the white noise intensity. In the case of the linear system (1) power $P_W = \gamma z_2 = D$. It should be stressed that the harvested power is independent of the system's properties. This result provides an upper boundary for the harvested power but does not show how a system can be adjusted to achieve its best performance. It happens because this result was derived for a white noise excitation, which is an unrealistic mathematical model of a physical random process with finite power and variance. If the excitation in (1) to be a much more realistic model, like the Ornstein–Uhlenbeck process (25) for example, then power of (1) can be obtained exactly analytically using the above approach:

$$P_\mu = \gamma z_2 = \frac{D(1+\gamma\mu)}{1+\gamma\mu+\omega^2\mu^2} = P_W \frac{1}{1+\dfrac{\omega^2\mu^2}{1+\gamma\mu}}$$

The derived expression tends to the above result as $\mu\to 0$ and the Ornstein–Uhlenbeck process tends to a white noise. Apparently the system power P_μ will always be smaller than that due to a white noise, as expected, for $\mu > 0$. Nevertheless formula for P_μ explicitly indicates how the system's parameters influence the power absorption, and therefore it can be used for designing a harvester. For instance, the above expression indicates that a system with a very low natural frequency ($\omega < 1$) will be much more effective compared with that with higher natural frequency, pointing out towards a low frequency energy harvesting direction.

Although it has been established that the amount of power a system can harvest is limited, it has been proved for the systems with an external excitation. The aim of this paper is to propose a novel method for the vibration-based energy harvesting. In this approach the vibratory system is still linear but it has a random time-varying parameter with specially selected characteristics. It allows significantly to increase the harvested power, beyond the above established limit, as can be seen later.

Let us consider now the linear oscillator with a random parametric excitation

$$\ddot{x} + \gamma\dot{x} + [\omega^2 + \sigma\xi(t)]x = \eta(t), \tag{4}$$

where $\xi(t)$ is a formally defined telegraphic noise, i.e. zero-mean ergodic Markov process with two state $\{-1, 1\}$ and the transition rate $\alpha/2$, $\sigma > 0$ is an intensity of the noise. The telegraphic noise is a simple but quite useful model of the stochastic process (see e.g. the books [8,9]. For the sake of simplicity we assume that the processes $\xi(t)$ and $\eta(t)$ are independent. Then the solution to Eq. (4) in terms of second-order moments can be obtained in two steps. The first step is an averaging over η and the second one is an averaging of the results from the first step over $\xi(t)$.

For the vector $\mathbf{u} = (E_\eta[x^2], E_\eta[\dot{x}^2], E_\eta[x\dot{x}])^T$ we obtain similar to (2) the differential equation in the matrix form

$$\dot{\mathbf{u}} = A\mathbf{u} + \xi(t)B\mathbf{u} + \mathbf{c}, \tag{5}$$

where $E_\eta[\cdot]$ is averaging over the white noise η, $\mathbf{c} = (0, 4D, 0)^T$,

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & -2\gamma & -2\omega^2 \\ -\omega^2 & 1 & -\gamma \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2\sigma \\ -\sigma & 0 & 0 \end{pmatrix}.$$

After averaging Eq. (5) over the process ξ and using results of Refs. [10,11] we get

$$\begin{aligned} \frac{\mathrm{d}E_\xi[\mathbf{u}(t)]}{\mathrm{d}t} &= AE_\xi[\mathbf{u}] + B\mathbf{u}_1 + \mathbf{c}, \\ \frac{\mathrm{d}\mathbf{u}_1(t)}{\mathrm{d}t} &= -\alpha\mathbf{u}_1 + A\mathbf{u}_1 + BE_\xi[\mathbf{u}], \end{aligned} \tag{6}$$

where $\mathbf{u}_1(t) = E_\xi[\xi(t)\mathbf{u}(t)]$. Therefore the set of six linear differential equations is obtained for the second-order moments of system (4). One can derive exact analytical expressions for the stationary moments solving the appropriate set of six linear