Journal of Sound and Vibration 333 (2014) 4711–4723

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Multi-scale analysis on nonlinear gyroscopic systems with multi-degree-of-freedoms

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ARTICLE INFO

Article history:
Received 29 November 2013
Received in revised form
2 April 2014
Accepted 2 May 2014
Handling Editor: D.J. Wagg
Available online 28 May 2014

ABSTRACT

A method of multiple scales is developed for n-degree-of-freedom weakly nonlinear gyroscopic systems. A general procedure is proposed to establish solvability conditions. The conditions have n different versions whose equivalence cannot be mathematically demonstrated. The procedure is applied to a 4-degree-of-freedom nonlinear gyroscopic system that is the 4-term Galerkin truncation of the governing equation of a pipe conveying fluid flowing in the supercritical speed. The investigation focuses on the primary external resonance in the first frequency ω_1 and the two-to-one internal resonance of the first two frequencies ω_1 and ω_2. The multi-scale analysis shows that the amplitude–frequency response curve in each of the first two modes has a peak bending to the left when $\omega_2 > 2\omega_1$, two peaks with the same height and the opposite bending directions when $\omega_2 = 2\omega_1$, and a peak bending to the right when $\omega_2 < 2\omega_1$. In all those cases, the 4 different versions of the solvability conditions yield same outcomes. The responses in the last two modes uninvolved in the resonances decay to zero exponentially. The numerical integration results are qualitative agreement with the analytical ones.

1. Introduction

Multi-degree-of-freedom gyroscopic systems are derived from the differential equations of rigid-body relative motions and the Galerkin truncations of translating or rotating continua. The terminology "gyroscopic", arisen in early studies on gyrodynamics, is now employed to indicate the existence of terms produced by the Coriolis acceleration component. Those gyroscopic terms are associated with skew-symmetric matrixes in differential equations of motions. The particular mathematical structure requires some specific approaches to analyze a gyroscopic system.

Although the modal solution to linear gyroscopic systems has been worked early [1] and the stability of linear gyroscopic systems has been widely studied [2], the investigations on approximate analytical approaches to vibration of gyroscopic systems are rather limited. Early works focused on linear problems, and only two examples will be mentioned in what follows. To avoid computational difficulties caused by complex quantities, Meirovitch and Ryland proposed a perturbation technique to determine the response of damped gyroscopic system via real computations based on the response of corresponding undamped gyroscopic system [3]. To calculate the response of time-dependent gyroscopic systems, Wickert

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http://dx.doi.org/10.1016/j.jsv.2014.05.005
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developed an asymptotic approach [4] based on the idea of Bogoliubov and Mitropolsky [5]. Nonlinear vibration of gyroscopic systems can also be treated via approximate analytical approaches. Nayfeh and Mook presented the method of multiple scales for free oscillations of a gyroscopic system with quadratic nonlinearities [6]. Ariaratnam and Namachchivaya used the method of averaging to explore bifurcation behavior of periodically perturbed nonlinear gyroscopic systems with the application in a rotating shaft carrying a heavy rotor [7]. Nagata1 and Namachchivaya investigated local and global bifurcations in the gyroscopic system consisting of unperturbed Hamiltonian system under symmetry-breaking and damping perturbations and applied the theoretical results to vibration of a rotating shaft [8]. Based on the Galerkin truncation of the governing equations of axially moving beams, Sze, Chen, and Huang formulated the incremental harmonic balance method to examine the forced periodic responses in the presence of internal resonance and their stabilities [9]. Treating transverse nonlinear vibration of axially moving beams, Chen, Huang, and Sze proposed the Lindstedt–Poincaré method for multi-degree-of-freedom gyroscopic systems and determined the steady-state responses in the primary resonance [10]. Motivated by vibration of pipers conveying fluid, Zhang and Chen developed the method of multiple scales for forced oscillations of a gyroscopic system with quadratic nonlinearities in the presence of internal resonance [11].

In the above-mentioned investigations on analytical approaches to nonlinear gyroscopic systems, all worked examples are with two-degree-of-freedoms. There is an essential difference between the two-degree-of-freedom and the higher-number-degree-of-freedom systems in the view of analytical approaches. As Nayfeh observed, the solvability condition in the method of multiple scales may take different forms, and only in two-degree-of-freedom systems, it can be mathematically proved that the different forms yield the same results [12]. The similar observation can be made in the applications of other analytical methods, for example, the multidimensional Lindstedt–Poincaré method [10]. So far, it is not clear if the different forms of the solvability condition are equivalent in higher-dimensional cases. In addition, although there has been the idea of method of multiple scales for 2-degree-of-freedom gyroscopic systems under time-independent nonlinear perturbations [6], time-dependent linear perturbations [12] or time-dependent nonlinear perturbations [11], there are neither general formulations of the method of multiple scales nor applications in systems with degree-of-freedom larger than 2. To address the lack of research in these aspects, the present work formulates a general framework of multi-scale analysis on nonlinear gyroscopic systems, and in the framework a four-degree-of-freedom system is treated to explore the equivalence of the different forms of the solvability condition.

The four-degree-of-freedom nonlinear gyroscopic system is derived from the 4-term Galerkin truncation of a nonlinear partial differential equation governing the forced vibration of a pipe conveying fluid [13]. Vibration of pipes conveying fluid is a challenging subject that has been investigated for many years and is still of interest nowadays [14–16]. Based on the 2-term Galerkin truncation, an interesting phenomenon was revealed in the presence of the external and the internal resonances. In the super-critical regime, with the increase of the fluid speed (the parameter appearing in a linear term), each amplitude–frequency curves in the first two modes are with a peak bending to the left, 2 peaks bending to the opposite directions, and finally a peak bending to the right [17]. The mechanism of the phenomenon is not well understood. However, it is associated with the modal interaction [18,19] due to internal resonances. The present work will examine if the phenomenon appears in the 4-term Galerkin truncation. If the phenomenon still exists, the equivalence of the different forms of the solvability condition will be checked in amplitude–frequency responses curves with a left-bending peak, two opposite-bending peaks, and a right-bending peak.

The paper is organized as follows. Section 2 proposes a general framework of multi-scales analysis on nonlinear gyroscopic systems. In Section 3, the general theory is applied to a specific example, forced vibration of pipes conveying fluid in the supercritical regime. The steady-state responses are derived from the solvability condition, and the different forms of the condition yield the same amplitude–frequency responses curves. The softening, the double-jumping, and the hardening characteristics revealed by the analytical results are supported by the direct numerical integrations. Section 4 ends the paper with concluding remarks.

2. General framework of the method of multiples scales

Consider an n-degree-of-freedom linear undamped gyroscopic system with a weak time-dependent disturbance. Assume that the system is specified by an n-dimensional vector of generalized coordinates $\boldsymbol{q}$. Then the system is defined by

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}}+G\dot{\boldsymbol{q}}+\boldsymbol{K}\boldsymbol{q}=\boldsymbol{\varepsilon}\,\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}},\tau)+\boldsymbol{O}(\varepsilon^2,\varepsilon^3,\ldots) \tag{1}$$

where $n\times n$ mass matrix $\boldsymbol{M}$ and $n\times n$ stiffness matrix $\boldsymbol{K}$ are symmetric ($\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}$, $\boldsymbol{K}=\boldsymbol{K}^{\mathrm{T}}$), $n\times n$ gyroscopic matrix $\boldsymbol{G}$ is skew-symmetry ($\boldsymbol{G}=-\boldsymbol{G}^{\mathrm{T}}$), ε denotes a small dimensionless parameter to indicate that the disturbance is weak, $\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \tau)$ stands for a nonlinear function vector with respect to $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and τ, and $\boldsymbol{O}(\varepsilon^2, \varepsilon^3,\ldots)$ represents the vectors whose all elements are in the same order as ε^2 or higher. In addition, $\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \tau)$ is explicitly periodic in τ with period $2\pi/\omega$.

Introduce two time scales $T_0=\tau$ and $T_1=\varepsilon\tau$. A perturbation solution is sought in the form

$$\boldsymbol{q}(\tau)=\boldsymbol{q}_0(T_0,T_1)+\varepsilon\boldsymbol{q}_1(T_0,T_1)+\boldsymbol{O}(\varepsilon^2,\varepsilon^3,\ldots) \tag{2}$$

Then

$$\dot{\boldsymbol{q}}_i=D_0\boldsymbol{q}_i+\varepsilon D_1\boldsymbol{q}_i,\ddot{\boldsymbol{q}}_i=D_0^2\boldsymbol{q}_i+2\varepsilon D_0D_1\boldsymbol{q}_i+\boldsymbol{O}(\varepsilon^2,\varepsilon^3,\ldots)\quad(i=1,2) \tag{3}$$