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Damage identification for beams in noisy conditions based on Teager energy operator-wavelet transform modal curvature

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ABSTRACT

Modal curvatures have been widely used in the detection of structural damage. Attractive features of modal curvature include great sensitivity to damage and instant determination of damage location. However, an intrinsic deficiency in a modal curvature is its susceptibility to the measurement noise present in the displacement mode shape that produces the modal curvature, likely obscuring the features of damage. To address this deficiency, the Teager energy operator together with wavelet transform is tactically utilized to treat modal curvature, producing a new modal curvature, termed the Teager energy operator-wavelet transform modal curvature. This new modal curvature features distinct capabilities of suppressing noise, canceling global trends, and intensifying the singular feature caused by damage for a measured mode shape involving noise. These features maximize the sensitivity to damage and accuracy of damage localization. The proposed modal curvature is demonstrated in several analytical cases of cracked pinned–pinned, clamped–free and clamped–clamped beams, with emphasis on characterizing damage in noisy conditions, and it is further validated by an experimental program using a scanning laser vibrometer to acquire mode shapes of a cracked aluminum beam. The Teager energy operator-wavelet transform modal curvature essentially overcomes the deficiency of conventional modal curvature, providing a new dynamic feature well suited for damage characterization in noisy environments. (The Matlab code for implementing Teager energy operator-wavelet transform modal curvature can be provided by the corresponding author on request.)

1. Introduction

Structural damage detection has been a research focus of increasing interest in mechanical, civil, aerospace, and military fields during the last few decades [1–9]. Damage detection relying on modal curvatures has been extensively discussed in the literature [10–13]. Modal curvature is the curvature of a displacement mode shape for a structure. In the case of a beam,

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its modal curvature can be interpreted as [10]

$$-\frac{M}{EI(x)} = W''(x) \approx \frac{W(x-h) - 2W(h) + W(x+h)}{h^2} \tag{1}$$

where M is the bending moment, $EI(x)$ is the bending stiffness, $W(x)$ is the displacement mode shape for the beam, and $W''(x)$ is the modal curvature (the two primes denote second-order differentiation of $W(x)$). $W''(x)$ is approximately obtained by the second-order central difference of $W(x)$ with sampling interval h. Damage, commonly represented by an alteration in $EI(x)$, can cause a change in $W''(x)$, which in turn manifests the status of the damage. Thus the use of modal curvature is theoretically feasible for characterizing damage in beams.

Modal curvatures allow not only the detection of damage in a structure but also determination of the location of damage with a high degree of accuracy [14–15]. Pandey [10] originally proposed the dynamic feature of modal curvature for use in damage detection in beams: the abrupt change of modal curvatures in the vicinity of a crack indicates the presence and location of the damage. In damage detection applications, modal curvature is computed by the second-order central difference of the displacement mode shape [10–14]. Despite its popularity in characterizing damage, the modal curvature has a noticeable drawback: it is susceptible to any slight noise existing in a displacement mode shape [15–17]. This susceptibility can be attributed to the fact that the second-order central difference used to generate a modal curvature considerably amplifies any slight noise in the mode shape [18]. The amplified noise easily masks features of damage, possibly frustrating damage detection. Several researchers [14–17] have made efforts to study strategies to deal with this deficiency of modal curvature. The resulting methods can be roughly categorized into two types: (1) the effect of measurement noise on modal curvature is diminished by taking the optimal sampling interval of a mode shape to generate the modal curvatures; and (2) the immunity of modal curvature to noise is improved by sophisticated signal processing techniques. Unfortunately, these methods either require a complex procedure to determine the optimal sampling interval or exhibit strong dependence on the selected signal processing method. An accurate and reliable strategy capable of overcoming this drawback of modal curvature is yet to be presented. Unlike previous studies, this study focuses on developing a new modal curvature based on the use of a wavelet transform (WT) [19,20] incorporating the Teager energy operator (TEO) [21,22]. The new modal curvature, termed TEO-WT modal curvature, features noise suppression, cancellation of global trends, and intensification of the local singular feature for the signal under inspection. These features are eminently suitable for characterizing damage in noisy conditions. The capabilities of the TEO-WT modal curvature are investigated in analytical cases of cracked beams. The applicability of the approach is validated by an experimental program using a scanning laser vibrometer (SLV) [23,24] to acquire mode shapes of an aluminum beam with a crack.

2. TEO-WT modal curvature

2.1. TEO

The discrete version of the TEO [21] was proposed by Kaiser [22] with the aim of representing the transient energy of a signal. Let x_n be a sequence of sampling points of a discretized cosine signal:

$$x_n = A\cos(\Omega n + \phi), \tag{2}$$

where n is the sampling number, ϕ is the initial phase and Ω is the digital frequency specified by $\Omega = 2\pi f/f_s$,with f being the analog frequency and f_s is the sampling frequency. The signal values at three successive points are

$$x_{n-1} = A\cos(\Omega(n-1)+\phi); \quad x_n = A\cos(\Omega n+\phi); \quad x_{n+1} = A\cos(\Omega(n+1)+\phi). \tag{3}$$

According to the trigonometric identities, we can obtain

$$x_n^2 - x_{n-1}x_{n+1} = A^2\sin^2(\Omega). \tag{4}$$

With the relation in Eq. (4), an algorithm to approximately calculate the point-wise energy E_n of a sole-component signal is given as

$$E_n = A^2\Omega^2 \approx A^2\sin^2(\Omega) = x_n^2 - x_{n-1}x_{n+1}. \tag{5}$$

Considering Eqs. (4) and (5), the TEO for a discrete sequence $f[n]$ is defined by

$$\Psi(f[n]) = f^2[n] - f[n-1]f[n+1]. \tag{6}$$

where Ψ denotes the TEO that calculates the approximate transient energy of $f[n]$.

Currently, the TEO is widely used in speech engineering as an effective nonlinear operator to treat the local singularity of a speech signal [22,25,26], where it behaves like a supplement to the linear Fourier transform that is suitable for the analysis of global characteristics [19]. As with local singularity characterization for a speech signal, the TEO in this study is utilized to characterize the abnormality of a mode shape that is caused by structural local damage.