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Fatigue damage estimation in non-linear systems using a combination of Monte Carlo simulation and the First Order Reliability Method

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ABSTRACT

For non-linear systems the estimation of fatigue damage under stochastic loadings can be rather time-consuming. Usually Monte Carlo simulation (MCS) is applied, but the coefficient-of-variation (COV) can be large if only a small set of simulations can be done due to otherwise excessive CPU time. The reason is that the fatigue damage estimation is very sensitive to the largest values from the simulations. The paper suggests the additional use of the First Order Reliability Method (FORM) to get a better estimation of the tail in the distribution of the estimated fatigue damage and thereby reducing the COV. For a specific example dealing with stresses in a tendon in a tension leg platform the COV is thereby reduced by a factor of three.

1. Introduction

Wave-induced loads on marine structures can lead to fatigue damage due to the induced time variation in the stresses. Hence, it is important to estimate the fatigue damage in the design phase. Usually the expected fatigue damage is determined by a rainflow counting method, e.g. Refs. [1–3] as it generally provides the best agreement with measurements. Alternative methods, e.g. the use of regular waves identified by a cycle counting method from a large number of simulations of irregular waves

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have been applied [4], in the fatigue damage evaluation. Thereby the time-consuming load and stress calculations are simplified considerably as only a set of regular waves now is used as input. The expected fatigue damage is afterwards simply obtained using the relevant SN-curve and Miners rule. Whereas this procedure can provide accurate results, the error induced using a counting procedure in the waves rather than in the resulting stresses is difficult to quantify.

The use of Monte Carlo simulations (MCS) for the stress analysis requires several simulations with irregular wave trains as input. Each simulation must have duration long enough to remove the influence of the arbitrary initial conditions and furthermore yielding a stable rainflow counting result. The mean value of the expected fatigue damage is finally obtained as the average value obtained from these simulations. With relative few simulations possible due to time-consuming load and stress calculations the average value will depend very much on the largest value found from the simulation implying generally a large coefficient-of-variation (COV). An alternative to rainflow counting is spectral methods, e.g. Refs. [15], but these will have the same problem with the uncertain tail in the stress distribution.

The First Order Reliability Method (FORM) has previously been shown to give good results for extreme wave load predictions, e.g. Refs. [5–9]. For average values the FORM analysis cannot be used as it is only accurate for threshold values with a low probability of exceedance. However, in the following the FORM procedure will be used in an attempt to improve the accuracy of MCS simulations of the expected fatigue damage and thereby reducing the COV of the estimation. As an example, the stresses in one of the tendons in a tension leg platform holding a wind turbine will be considered [10]. Two different load scenarios are considered, one where the irregular waves are determined by linear theory and one where the waves also include the second order contributions, [5]. The last case requires significant computational effort and is therefore a good example for the proposed procedure. A stationary sea state is assumed, but non-stationary cases can be dealt with as usual by a weighting procedure using the relevant scatter diagram. Here it should be mentioned that the FORM analysis does not need to be recalculated if only the significant wave height is changed as the FORM reliability index is strictly inversely proportional to the significant wave height, e.g. Ref. [8].

2. Fatigue damage estimation

For moderate, stationary sea states the wave elevation can be considered as Gaussian distributed, whereas for more severe sea states non-linearity in the wave model must be incorporated. A second-order stochastic wave model was described in Ref. [5] and can easily be included as it does not involve additional stochastic parameters. Linear, long-crested waves are thus the basic input process. Hence, the normal distributed wave elevation $H(x,t)$ as a function of space x and time t can be written in discretized form as

$$\begin{aligned} H(x,t) &= \sum_{i=1}^{m}\left(u_i c_i(x,t) + \overline{u}_i \overline{c}_i(x,t)\right) \\ c_i(x,t) &= \sigma_i \cos(\omega_i t - k_i x) \\ \overline{c}_i(x,t) &= -\sigma_i \sin(\omega_i t - k_i x) \\ \sigma_i^2 &= S(\omega_i) d\omega_i \end{aligned} \tag{1}$$

where the variables $\underline{u} = \{u_i, \overline{u}_i\}_{i=1,2,..,m}$ are statistically independent, standard normal distributed variables and where the dispersion relation $gk_i = \omega_i^2$ couples the m discrete frequencies ω_i and wave numbers k_i applied. Here g is the acceleration of gravity. Furthermore, $S(\omega)$ is the wave spectrum and $d\omega_i$ the increment between the frequencies applied. From this wave model a time domain analysis using a proper hydrodynamic model including also second order wave components can be performed to obtain the desired response, here stresses.

The expected fatigue damage D per unit time is then determined by a rainflow counting (RF) procedure for the stress signal:

$$D(\underline{u}) = \frac{1}{T} RF[t_0, t_0 + T | \underline{u}] \tag{2}$$