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On wave propagation in gradient poroelasticity

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ABSTRACT

Using the governing equations of motion of a fluid- saturated poroelastic medium including micro-stiffness (for the solid and the fluid) and micro-inertia (for the solid) effects, propagation of plane harmonic waves are studied in the low and high frequencies range. The study involves both dilatational and rotational waves and focuses on the micro-stiffness and micro-inertia effects on the dispersion and attenuation of these waves.

1. Introduction

The quasi-static and dynamic theories of linear poroelasticity due to Biot [1,2] have found many applications in geotechnical engineering, bioengineering and materials science and engineering. A comprehensive review on the subject of poroelastodynamics involving linear models and analytical and numerical methods of solution up to 2009 has been reported by Schanz [3].

For some classes of materials like granular ones, which possess a natural microstructure, classical linear elasticity cannot take into account microstructural effects and resort should be made to higher order theories that do take into account those effects. A general linear theory of elasticity with microstructural effects (both of the micro-stiffness and micro-inertia type) has been developed by Mindlin [4]. The simplest possible version of that theory has two elastic constants (internal length scale) in addition to the two classical elastic modili, which express the microstructural (micro-stiffness and micro-inertia) effects in a microscopic manner. Using that simple theory, called gradient elastic theory, various wave propagation problems have been solved (e.g. [5,6]) and the microstructural effects on dispersion curves have been assessed. Microstructural effects in fluid-saturated poroelastic media have been studied by Berryman and Thigpen [7] through micro-inertia terms and Aifantis [8] and Vardoulakis and Aifantis [9] through second order gradients for densities and fluid pressure. More recently, Sciarra et al. [10] and Madeo et al. [11] developed a poroelastic theory involving gradient effects (micro-stiffness of solid) and studied the one – dimensional consolidation (quasi-static) problem of soil mechanics. Recently, Papargyri-Beskou et al. [12] introduced second order gradient of strain in the stress-strain relation for the solid component of a fluid-saturated poroelastic medium and stadied the gradient effect on the dynamic column problem of soil mechanics. Finally, Papargyri-Beskou et al. [13] developed a thee-dimensional poroelasticity theory with micro-stiffness effects for both the solid and the fluid and micro-inertia effects for the solid and studied wave dispersion and attenuation in the low frequency range.

In this Note, the work in [13] is extended to the high frequency range by following Biot [2] and the effects of the microstructural parameters on wave dispersion and attenuation in poroelastic media are assessed.

2. Wave propagation analysis

The governing equations of motion of a fluid-saturated poroelastic medium including microstructural effects as derived in [13] with the aid of the theory of mixtures have the form

$$(1-g^2\nabla^2)[(\lambda+\mu)u_{\kappa,\kappa i}+\mu\nabla^2 u_i] = -\frac{\beta\nu_f n}{K}(\dot{u}_i^f-\dot{u}_i)+\bar{\rho}_s\ddot{u}-\bar{\rho}_s h^2\nabla^2\ddot{u}_i \tag{1}$$

$$-\beta(1-\theta^2\nabla^2)p_{,i} = n\rho_f\ddot{u}_i^f+\frac{\beta\nu_f n}{K}(\dot{u}_i^f-\dot{u}_i) \tag{2}$$

$$\beta\dot{u}_{i,i}+n\dot{u}_{i,i}^f+(\gamma+\delta n)(1-\theta^2\nabla^2)\dot{p}=0 \tag{3}$$

The above equations form a system of 3+3+1=7 partial differential equations with 7 unknowns, that is, three solid displacements u_i, three fluid displacements u_i^f and one porewater

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pressure p. The coefficients g and h with dimensions of length correspond to micro-stiffness and micro-inertia of the solid, while θ, also with dimensions of length, to micro-stiffness of the fluid. When g=h=θ=0 there are no microstructural effects. Furthermore, λ and μ are the classical Lame constants, ν_f is the dynamic viscosity of the fluid, K the Muskat permeability, n the porosity, β a coefficient expressing the solid deformability affecting the fluid flow, γ is a coefficient measuring the compressibility of the porous medium, δ is the fluid compressibility and $\bar{\rho}_s = \rho_s(1-n)$ and $\bar{\rho}_f = n\rho_f$ with ρ_s and ρ_f being the mass density of the solid and fluid, respectively. Finally it should be mentioned that indicial notation is used everywhere, a comma denotes differentiation with respect to space, overdots denote differentiation with respect to time and repeated indices indicate summation.

Application of the divergence and the rot (curl) operators on Eqs. (1) and (2) and employment of Eq. (3) results in the dilatation equations of motion [13].

$$(\lambda+2\mu)(1-g^2\nabla^2)\nabla^2\varepsilon = \frac{\beta\nu_f}{K}[(\beta+n)\dot{\varepsilon}+(\gamma+\delta n)(1-\theta^2\nabla^2)\dot{p}] + \bar{\rho}_s(1-h^2\nabla^2)\ddot{\varepsilon} \tag{4}$$

$$\beta(1-\theta^2\nabla^2)\nabla^2 p = \rho_f\beta\ddot{\varepsilon} + \rho_f(\gamma+\delta n)(1-\theta^2\nabla^2)p + \frac{\beta\nu_f}{K}\left[(\beta+n)\dot{\varepsilon}+(\gamma+\delta n)(1-\theta^2\nabla^2)\dot{p}\right] \tag{5}$$

and the rotational equations of motion [13].

$$\mu(1-g^2\nabla^2)\nabla^2\boldsymbol{\omega} = n\rho_f\ddot{\boldsymbol{\omega}}^f + \bar{\rho}_s(1-h^2\nabla^2)\ddot{\boldsymbol{\omega}} \tag{6}$$

$$\rho_f\ddot{\boldsymbol{\omega}}^f + \frac{\beta\nu_f}{K}\dot{\boldsymbol{\omega}}^f - \frac{\beta\nu_f}{K}\dot{\boldsymbol{\omega}} = 0 \tag{7}$$

where

$$\varepsilon = u_{i,i},\quad \boldsymbol{\omega} = \frac{1}{2}\nabla\times\mathbf{u},\quad \boldsymbol{\omega}^f = \frac{1}{2}\nabla\times\mathbf{u}^f \tag{8}$$

Consider first plane harmonic waves in the x-z plane propagating along the z direction in an infinitely extended medium moving according to Eqs. (4) and (5). Thus [13].

$$\varepsilon = A\exp[i(\omega t-\Lambda z)],\quad p = B\exp[i(\omega t-\Lambda z)] \tag{9}$$

where A and B are amplitudes, ω, Λ the complex wavenumber and $i=\sqrt{-1}$. It can be shown that [13].

$$(\varepsilon, p) = (A, B)e^{-\Delta_p z}e^{i(\omega t-k_p z)} \tag{10}$$

where k_p and Δ_p are the wavenumber and attenuation coefficient, respectively, for dilatation or P-waves, connected via Λ as $\Lambda = k_p(\omega) - i\Delta_p(\omega)$ with the Λ's obtained as the roots of an algebraic equation of the 8th order [13]. The phase velocity C_p of P-waves is then obtained by $C_p = \omega/k_p(\omega)$.

Consider now plane harmonic waves in the x-z plane propagating along the z direction in an infinitely extended medium moving according to Eqs. (6) and (7). Thus [13]

$$\boldsymbol{\omega} = \mathbf{A}\exp[i(\omega t-\bar{\Lambda}z)] \qquad \boldsymbol{\omega}^f = \mathbf{B}\exp[i(\omega t-\bar{\Lambda}z)] \tag{11}$$

where **A** and **B** are vector amplitudes and $\bar{\Lambda}$ the complex wavenumber. It can be shown that [13]

$$(\boldsymbol{\omega}, \boldsymbol{\omega}^f) = (\mathbf{A}, \mathbf{B})e^{-\Delta_s z}e^{i(\omega t-k_s z)} \tag{12}$$

where k_s and Δ_s are the wavenumber and attenuation coefficient, respectively, for rotational or S-waves, connected via $\bar{\Lambda}$ as $\bar{\Lambda} = k_s(\omega) + i\Delta_s(\omega)$ with $\bar{\Lambda}$ obtained as the roots of an algebraic equation of the 4th order [13]. The phase velocity C_s of S-waves is then obtained by $C_s = \omega/k_s(\omega)$.

3. Dispersion and attenuation curves

The physical quantities of interest here are the frequency-dependent velocities of propagation C_{p1}, C_{p2}, C_s and the corresponding attenuation coefficients Δ_{p1}, Δ_{p2}, Δ_s. The variation of these quantities with frequency is presented here for various combinations of the microstructural parameters g, h and θ. All quantities are normalized. Thus, C_{p1}, C_{p2} are normalized by $C_{pf} = [(1/\delta)/\rho_f]^{1/2}$, C_s by $C_{rs} = (\mu/\bar{\rho}_s)^{1/2}$, ω is normalized by $\omega_{rp} = \beta\nu_f/K\bar{\rho}_s$, Δ_{p1}, Δ_{p2} are normalized by ω_{rp}/C_{rp} with $C_{rp} = [(\lambda+2\mu)/\bar{\rho}_s]^{1/2}$, and Δ_s is normalized by ω_{rs}/C_{rs} with $\omega_{rs} = \beta\nu_f/K\rho_f$.

According to Biot [2], Poiseuille flow in the pores breaks down for frequencies higher than $\omega_t = \pi\nu_f/4\rho_f d^2$, where d is the pore diameter. In the low frequency range ($\omega \le \omega_t$), ν_f is constant, while in the higher frequency range, ν_f is replaced by $\nu_f F(\omega)$, where $F(\omega)$ is a frequency correction factor expressed in terms of Bessel-Kelvin functions. The work in [13] was restricted to the low frequency range. The present Note extends that work to the high frequency range. There is also an upper bound ω_u for the frequency in the higher range to ensure that the medium is still continuum, i.e., the value at which the wavelength becomes of the order of the pore size. Thus, $\omega_u = 2\pi C/d$ where C is the phase velocity of the S wave for the soil-fluid mixture computed as $C = (\mu/(\bar{\rho}_s+\bar{\rho}_f))^{1/2}$. Following Beskos et al. [14], the correction factor F(ω) is expressed as

$$\omega < \frac{4\nu_f}{d^2\rho_f},\quad F(\omega) = \left(1+\frac{(d/2)^4\rho_f^2\omega^2}{1.152\nu_f^2}\right) + i\left(\frac{(d/2)^2\rho_f\omega}{24\nu_f}\left(1-\frac{(d/2)^4\rho_f^2\omega^2}{1.440\nu_f^2}\right)\right) \tag{13}$$

$$\frac{4\nu_f}{d^2\rho_f} \le \omega \le \frac{400\nu_f}{d^2\rho_f},\quad F(\omega) = \sum_{r=0}^{\infty}\frac{(-r)^r}{(r!)^2}\left(\frac{i^{3/2}d\rho_f\omega^{1/2}}{4\nu_f}\right)^{2r} \tag{14}$$

$$\omega > \frac{400\nu_f}{d^2\rho_f},\quad F(\omega) \approx \left\{\left(\frac{d}{8\sqrt{2}}\left(\frac{\omega\rho_f}{\nu_f}\right)^{1/2}\right)\left[1+\left(\frac{3\sqrt{2}}{d}\left(\frac{\nu_f}{\omega\rho_f}\right)^{1/2}\right)+\frac{15\nu_f}{2d^2\rho_f\omega}-\frac{135\nu_f^2}{2d^4\rho_f^2\omega^2}\right]\right.$$
$$\left. i\left\{\left(\frac{d}{8\sqrt{2}}\left(\frac{\omega\rho_f}{\nu_f}\right)^{1/2}\right)\left[1-\frac{15\nu_f}{2d^2\rho_f\omega} - \frac{15\sqrt{2}}{d^3}\left(\frac{\nu_f}{\omega\rho_f}\right)^{3/2} - \frac{135\nu_f^2}{8d^4\rho_f^2\omega^2}\right]\right\} \tag{15}$$

The numerical results of this section have been obtained on the basis of the values of the coefficients shown in Table 1 of [13], which correspond to a fully-saturated poroelastic sandstone. For these values one has $\omega_t = 7.85\times10^5$ rad/s, $\omega_{rp} = 4.352\times10^5$ rad/s and $\omega_{rs} = 8.838\times10^5$ rad/s. Thus, the low frequency results are valid for $\omega \le \omega_t$ or for $\widehat{\omega} = \omega/\omega_{rp} \le \widehat{\omega_t} = 1.804$ and $\widehat{\omega} = \omega/\omega_{rs} \le \widehat{\omega_t} = 0.888$ for P and S waves, respectively. Finally, $\omega_u = 140\times10^5$ sec^{-1} and hence $\widehat{\omega_u} = \omega_u/\omega_{rp} = 32.1738491$ and $\widehat{\omega_u} = \omega_u/\omega_{rs} = 15.840$ for P and S waves, respectively.

Using the above numerical data, dispersion and attenuation curves for the two P-waves and the S-waves of the considered gradient poroelastic medium have been obtained for the whole frequency range (low and higher) thereby completing the work in [13], which is restricted only to the low frequency range, i.e., up to $\widehat{\omega_t}$.

Figs. 1 and 2 present dispersion curves for the first (fast) P wave