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Technical Note

The energy factor of systems considering multiple yielding stages during ground motions

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ABSTRACT

This communication focuses on the energy factors of systems considering multiple yielding stages during ground motions. Based on an improved energy-balance model, the energy balance of systems is re-established and the energy factor is derived, considering the multiple yielding stages. To comprehensively investigate the influence of yielding stages and critical parameters on the energy factor, nonlinear dynamic analyses of single-degree-of-freedom systems are performed based on the validated numerical tool. Representative numerical evaluations are presented for different combinations of parameters denoting the yielding stages. The results indicate that the energy factor is significantly influenced by the yielding stages of systems. The results of this study are instructive for the calculation of the energy factor of systems showing multiple yielding stages during ground motions, and can be helpful to improve the current procedures based on the energy-balance concept.

1. Introduction

Since initiated by Housner [1], the energy-balance concept has been extended to structural evaluation and design in earthquake engineering for its conceptual simplicity and improved accuracy. Many research works in past decades also indicate that the feature of energy is a better index [2–6] as it captures the essence of behavior of systems during ground motions. In recent years, to consider the inelasticity of systems, a modified energy balance equation was established based on the elasto-plastic single-degree-of-freedom (SDOF) system [7]. Specifically, an energy factor considering the interaction of ground motion properties and systematic nonlinearity was introduced to consider the energy equilibrium. Based on this modification, extensive investigations have been made, and the derived energy factor has been applied in plastic design and evaluation of various structures [8–14]. With the energy factor and the classic Housner's equation, the demand during ground motions can be computed to design or evaluate the seismic capacity of the system. For instance, based on the energy factor of an elasto-plastic system, Sahoo and Chao [8] derived the design base shear for buckling-restrained braced frames and developed a design procedure. Jiang et al. [9] used the energy factor to determine the energy demand of systems and developed an energy-based multi-mode pushover analysis procedure. It should be noted that although these procedures are rational as they lead to reasonable accuracy, they were established on the elasto-plastic idealization which might conceal some real features of systems, especially for some innovative structures showing multiple yielding stages [11–13,15–18]. In this regard, it is more instructive to involve the yielding stages and to evaluate their effects when constructing the energy-balance equation. An improved model is still of significance for practical applications.

The objective of this communication is to investigate the energy factor considering multiple yielding stages. First, the energy balance is re-established, and the energy factor is derived, involving the multiple yielding stages. Based on the derived theory, a large amount of nonlinear dynamic analyses are performed to investigate the influence of multiple yielding stages on the energy balance during ground motions. The results are compared with conventional models, and the feasibility of the results is also validated.

2. The improved energy-balance concept and energy factor considering multiple yielding stages

By associating an inelastic system with the corresponding elastic system, the energy-balance equation of the inelastic SDOF system [7,9] can be established with the monotonic pushover skeleton curve and given by

$$\gamma\left(\frac{1}{2}MS_v^2\right) = E_e + E_p \tag{1}$$

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where γ, M, S_v, E_e and E_p are the energy factor, the mass of the system, the pseudo-velocity, the elastic energy and the plastic energy, respectively (Fig. 1). The energy factor essentially denotes the ratio of the energy absorbed by an inelastic system to that of the corresponding elastic system under monotonic loading. The cumulative effect is indirectly considered, and this equation has been applied in extensive research works [7–14], which is considered to be rational.

Practically, systems may exhibit the feature of multiple yielding stages as shown in Fig. 1, and this feature is remarkable especially for systems such as the damage-control systems with energy dissipation devices [15–17] and recently proposed innovative systems considering the performance-based design [11–14,18]. In essence, this feature is caused by yielding sequences of components in systems and has been observed by the research works of specific systems [11–13,15–18]. In this context, the ductility factor defined as the ratio of the maximum displacement to the first yield displacement [9,11,13] might only evaluate the maximum inelastic deformation, and the influence of the multiple yielding stages is neglected. It is also noted that the definition of ductility may be various in different research works and codes [19,20], but it is, in general, an equivalent index that cannot take the yielding stages into consideration explicitly.

To quantify the influence of yielding stages, the sequence factor is defined and given by

$$\zeta_{i-1} = \frac{\delta_{yi}}{\delta_{y1}} \tag{2}$$

where the δ_{y1} and δ_{yi} are defined as the first yield displacement and any expected target displacement denoting the post-yielding stages, respectively. The ductility factor (μ_s) is defined as the ratio of maximum displacement to the first yielding displacement (Fig. 2). Accordingly, the structural nonlinearity can be considered based on sequence factors, ductility factors, and the corresponding post-yielding stiffness ratio (α_i) in any stage, as illustrated in Fig. 2. And the energy-balance equation of the system can be established more accurately.

For a system with multiple stages of yielding, with a selected target displacement the absorbed energy calculated by the covered area of the skeleton pushover curve (Figs. 1 and 2) can be determined as

$$E_a = \boldsymbol{\lambda}_1^T \boldsymbol{\phi} \boldsymbol{\lambda}_2 \left(\tfrac{1}{2} V_{y1} \delta_{y1}\right) \tag{3a}$$

$$\boldsymbol{\lambda}_1 = [1, \zeta_1 - \zeta_0, \zeta_2 - \zeta_1, \ldots, \zeta_{n-1} - \zeta_{n-2}, \mu_s - \zeta_{n-1}]^T \tag{3b}$$

$$\boldsymbol{\lambda}_2 = [2\mu_s - 1,\ 2\mu_s - \zeta_1 - \zeta_0,\ 2\mu_s - \zeta_2 - \zeta_1,\ \ldots,\ 2\mu_s - \zeta_{n-1} - \zeta_{n-2},\ \mu_s - \zeta_{n-1}]^T \tag{3c}$$

$$\boldsymbol{\phi} = \mathrm{diag}[1, \alpha_1, \alpha_2, \ldots, \alpha_n] \tag{3d}$$

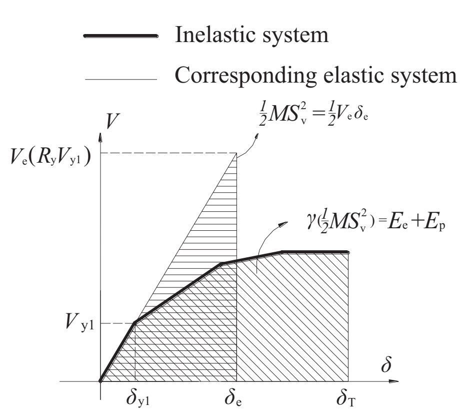

Fig. 1. The energy balance concept of systems.

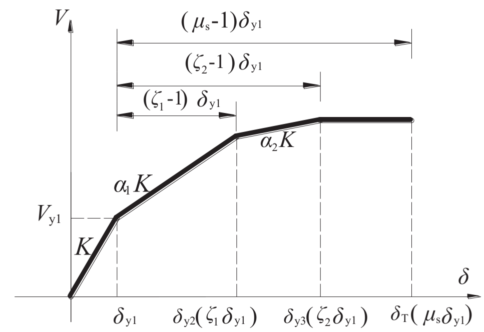

Fig. 2. The definition of sequence factor for systems with multiple yielding stages.

where E_a is the absorbed energy of the inelastic system with multiple yielding stages; α_i is the post-yielding stiffness ratio of the ith stage in the skeleton pushover curve; ζ_i is the ith sequence factor ($\zeta_0 = 1$); μ_s is the ductility factor; V_{y1} is the first yield strength of the system and δ_{y1} is the first yield displacement of the system.

On the other hand, the absorbed energy of the corresponding elastic system during ground motion can be calculated as

$$E_{ae} = \tfrac{1}{2} M S_v^2 = \tfrac{1}{2} V_e \delta_e \tag{4}$$

where E_{ae} is the absorbed energy of the corresponding elastic system; V_e is the maximum strength of the corresponding elastic system and δ_e is the maximum displacement of the corresponding elastic system.

Based on the definition stated above, following the procedure presented by Leelataviwat et al. [7], the energy factor considering multiple yielding stages is derived and given by

$$\gamma = \frac{\boldsymbol{\lambda}_1^T \boldsymbol{\phi} \boldsymbol{\lambda}_2}{R_y^2(T; \zeta_1, \zeta_2, \ldots, \zeta_{n-1}, \mu_s; \alpha_1, \alpha_2, \ldots, \alpha_n)} \tag{5a}$$

$$R_y = \frac{V_e}{V_{y1}} \tag{5b}$$

R_y is the strength reduction factor. It should be noted that the value of R_y still depends on the interaction effect of structural hysteretic behavior and ground motions. For systems exhibiting significant post-yielding stiffness such as damage-control systems [15–17], Eq. (5a) comes down to Eq. (6a). For systems exhibiting tri-linear behavior [12,14], Eq. (5a) then comes down to Eq. (6b).

$$\gamma = \frac{2\mu_s - 1 + \alpha_1(\mu_s - 1)^2}{R_y{}^2(T;\ \mu_s;\ \alpha_1)} \tag{6a}$$

$$\gamma = \frac{2\mu_s - 1 + \alpha_1(\zeta_1 - 1)(2\mu_s - \zeta_1 - 1) + \alpha_2(\mu_s - \zeta_1)^2}{R_y{}^2(T;\ \zeta_1,\ \mu_s;\ \alpha_1,\ \alpha_2)} \tag{6b}$$

Since the yielding stages change the nonlinear dynamic property of the system, it will correspondingly influence the energy demand of ground motions as indicated by the energy factor. Essentially, the yielding stages will lead to the transformation of the energy-balance mode, and the difference of energy balance mode will be reflected by structural responses such as displacement, force and other critical parameters.

In view of the essence of structural response during ground motions, it should be noted that it is not the total energy of an earthquake that determines the damage of a structure, but the rate with which this energy arrives and shakes the structure is essential [21,22]. Radically, this issue reveals that both the cumulative response and the peak response are significant to identify the structural damage and behavior. In essence, the survival of a structure subjected to an earthquake event is determined by