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# Stochastic finite element analysis of plates with the Certain Generalized Stresses Method

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ABSTRACT

The Certain Generalized Stresses Method (CGSM) takes into account variability in static finite element analysis. The CGSM is dedicated to thin-walled structures: bars, beams, plates and shells. The objective of this paper is to present and evaluate a methodology based on the CGSM, for the static finite element analysis of plates with variability. The CGSM is a non-intrusive method that requires only one finite element analysis with some load cases to calculate the variability of mechanical quantities of interest. The statistical results: mean value, standard deviation and distribution are obtained by Monte Carlo simulations, using a semi-analytical formula. Two examples are treated: a square plate with a circular hole under tension and a simply supported square bending plate under uniformly distributed load. The method provides results of good quality and is very economical from a computational time point of view.


## 1. Introduction

The mechanical behavior of structures always involves some uncertainty. Sometimes the effects of uncertainties may be large. Consequently, in the context of finite element analysis of structures, it is necessary to take uncertainty into account. During the last two decades much research has been done to develop and apply different approaches to typical structural mechanics examples. Nevertheless, this subject is still in progress and a lot of works currently deal with improvements to solve the uncertainty issue. Indeed these methods may be very expensive from a computational time point of view, in particular for large finite element models.

The stochastic finite element methods can be classified in two general categories: probabilistic approaches and possibilistic ones. In a probabilistic approach the input parameters are described by statistical distributions. The objective is to predict the statistics of output quantities. In possibilistic approaches, only the bounds on input parameters are defined. The objective is to calculate the bounds of the output quantities. Stefanou [1] distinguishes between three types of probabilistic methods: the Monte Carlo simulation [2–9], the perturbation method [10–17], and the spectral stochastic finite element method [18–26]. In the same way Moens and Hanss [27] distinguish between two types of possibilistic methods: the interval finite element method [28–31] and the fuzzy finite element method [32–34]. A lot of variants have been proposed, involving namely combinations of methods mentioned above. For example, Argyris et al. [35], Stefanou and Papadrakakis [36], Noh et al. [37–40] and Shang and Yun [41] use the spectral representation method for the description of random fields in conjunction with Monte Carlo simulation.

Taking into account uncertainty in finite element analysis of plate and shell structures, is also a research issue. In 1998, Graham and Deodatis [18] propose the spectral method for stochastic plate bending problems where the elastic modulus is defined by a homogeneous stochastic field. Further, Rahman and Rao [10] and Falsone and Impollonia [15] apply the perturbation method for the uncertain analysis of bars and membrane plates. In 2005, Noh develops the weighted integral method for Mindlin bending plates with uncertainty of elastic modulus and plate thickness [37] or with uncertainty of multiple material properties [38]. In 2006, Sachdeva et al. [19] study membrane plates and the settlement of a foundation with the spectral method. In 2007, Han et al. [34] use the wavelet-based stochastic finite element method for thin plate examples. Some research is also dedicated to uncertainty of composite plates. Noh and Park [17] study the influence of spatial randomness of Poisson's ratio. Antonio and Hoffbauer [11], Pandit et al. [13] and Chen and Soares [21] take into account uncertainty of several material properties.

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In 2012, Lardeur et al. [8] propose a new method called the Certain Generalized Stresses Method (CGSM), based on a mechanical assumption. The CGSM is dedicated to thin-walled structures: bars, beams, plates and shells. In Lardeur et al. [8], the main ideas of the CGSM are given and the method is applied to bar and beam structures. In this paper the CGSM is developed for the static analysis of plates with uncertain material or geometric properties represented by random fields.

In Section 2 the theoretical aspects of the CGSM for plates are presented. First the principle of the method is described. Then the meshing issue, in the context of uncertain finite element analysis, is discussed. The formulation of the CGSM is given for membrane plates and bending plates. The bending case is presented for thin and thick plates, without or with transverse shear effects respectively. Then the approach used to take into account input parameters defined by random fields is described. Finally error criteria to assess the CGSM are presented. Section 3 deals with examples. The first example is a square membrane plate with a circular hole under tension. The second example is a square bending plate subjected to a uniformly distributed load. In Section 4 some conclusions are drawn.

## 2. Theoretical aspects of the Certain Generalized Stresses Method

### 2.1. Principle of the Certain Generalized Stresses Method

The flowchart of the CGSM is shown in Fig. 1. The process described in this figure is valid for all types of thin-walled structures: bars, beams, plates and shells. The CGSM is based on the assumption that the generalized stresses are independent of the uncertain parameters. This assumption is strictly met for statically determinate structures and leads to exact results for this class of applications. This assumption is also met exactly if the perturbation due to uncertainty is uniform throughout the structure that is to say if all the terms of the stiffness matrix are multiplied by the same coefficient. For example, this is the case if the elasticity modulus is uncertain and if only one uncertain parameter is considered over the whole structure. Of course this case is trivial because it reduces the problem to one where the system properties vary homogeneously. In reality mechanical structures are generally statically indeterminate and the number of parameters is generally larger than one. Consequently the objective of this study is to assess the relevance of the mechanical assumption described above, for the general case, when the CGSM assumption is not exactly met. Thanks to this assumption, only one finite element run with some load cases, in the nominal configuration, is necessary to calculate the generalized stresses. It is then possible to calculate the strain energy of the system for all values of uncertain parameters without further finite element analysis. The displacement of a point of the structure is evaluated using Castigliano's theorem. The derivative of the strain energy with respect to the possible force applied at the point of interest leads to an expression of the displacement at this point. By using this expression, a Monte Carlo simulation is performed to calculate the mean value, standard deviation and distribution of the displacement. For some types of structures, the mean value and the standard deviation can also be obtained analytically. The CGSM is compatible with any statistical distribution. The CGSM is a non intrusive method and it is compatible with the use of any standard finite element software. In this study, Abaqus [44] has been used to treat the examples.

Finally, the CGSM can be considered as a post-treatment of one standard finite element calculation. To calculate the variability at $n$ points, one finite element analysis with $2n$ load cases at most are needed. This finite element analysis is performed in the nominal configuration, so for the different load cases the stiffness matrix is always the same and consequently, this matrix has to be inverted only once. This is an advantage from the computational time point of view.

### 2.2. Influence of the spatial discretization

Spatial discretization is an important issue in the finite element analysis of structures, particularly in presence of uncertainties. Using an optimal finite element mesh leads to an increase of the efficiency and to a reduction of the computational time. In an approach with variability, the objective is to find the optimal mesh for accurate calculation of statistical quantities of the results observed (displacements, strains...). In particular, the convergence

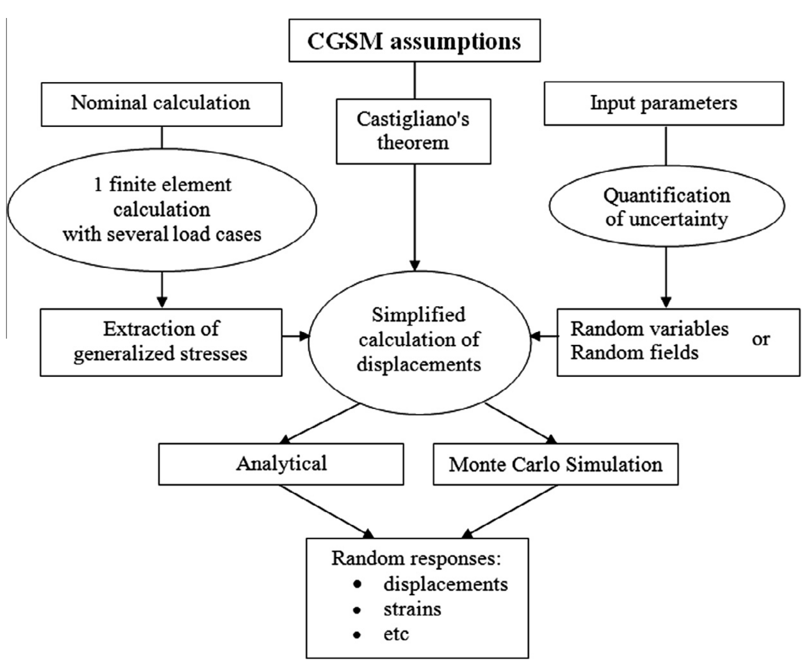


**Fig. 1.** Principle of the CGSM method for calculating variability.