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Probabilistic framework for assessing maximum structural response based on sensor measurements

Iris Tien [a,*], Matteo Pozzi [b], Armen Der Kiureghian [c]

[a] School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0355, USA
[b] Department of Civil and Environmental Engineering, Carnegie Mellon University, Pittsburgh, PA 15213-3890, USA
[c] Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720-1710, USA

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ABSTRACT

A probabilistic framework for Bayesian inference combined with extreme values of Gaussian processes is proposed to assess the maximum of the response of an uncertain structure instrumented with sensors and subject to a stochastic load. The framework is applied to the analysis of the inter-story drift of a multi-story shear-type building under seismic hazard using measurements collected by accelerometers. A cascade of two dynamic systems is proposed to model the stochastic ground motion and the response of the structure. We present an approximate analytical solution to estimate the distribution of the maximum response, and verify the accuracy and limitations of this solution against simulation results. Finally, robustness of the proposed framework to system uncertainties, including uncertainties in the structural characteristics, ground characteristics, and input motion parameters, is investigated.

1. Introduction

Monitoring of instrumented civil structures and infrastructure is becoming ubiquitous, as sensors continue to decrease in price and increase in capability. Structural health monitoring (SHM) methods have been developed to both improve the data collected through the development of new sensor devices, as well as facilitate how this data is used to learn about the system. The focus of this paper is on the latter, in how we process sensor data to perform inference on the structural response.

Monitoring systems provide real-time measurements on the dynamic response of structures during extreme events. Information about the structural model and the stochastic load can be integrated into the data processing to probabilistically evaluate features relevant to the post-event condition assessment, such as extreme values of key structural responses.

To do this task, in this paper we propose a framework based on the Kalman smoother for Gaussian linear systems and extreme value analysis of Gaussian processes. The objective is to accurately assess the maximum of the response of a linear structure under stochastic excitation by processing noisy sensor measurements. We apply the framework to the estimation of the inter-story drift of a multi-story, shear-type building under seismic hazard using information from measurements of accelerometers placed at selected floors of the building. We develop a cascade of two systems, modeling the seismic ground motion and the vibrating structure. We derive an analytical solution to estimate the distribution of the peak response, conditioned on the measurements, and results from this solution are compared with those obtained from Monte Carlo simulations. Finally, we show the proposed probabilistic framework to be robust to system uncertainties, including uncertainties in the structural characteristics, ground characteristics, and input motion parameters. This work informs decision making in the management of structures subject to seismic hazard and for the development and design of smart SHM systems.

2. Background and related work

For Gaussian linear models, the Kalman Filter (KF) [1] and Kalman Smoother (KS) [2] can be used to estimate the system state for dynamically evolving systems by processing sparse measures of the system response. While the KF algorithm computes the posterior probability of the system given past and present measurements, the KS algorithm allows, after an event, to compute the posterior distribution with respect to all measurements collected even after the time at which the state is being evaluated. The KF and KS allow computation of not only the marginal probability of

* Corresponding author at: School of Civil and Environmental Engineering, Georgia Institute of Technology, 790 Atlantic Drive, Atlanta, GA 30332-0355, USA.
E-mail address: itien@ce.gatech.edu (I. Tien).

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system state at each time, but also to sample trajectories. The reader is referred to the texts [3,4] for treatment of the KF and KS models.

As the KF and KS perform probabilistic analysis of a dynamic system, they are ideal for structural health monitoring (SHM) applications, where observations of a structure are used to characterize and assess the state of the structure over time. In this study, we are interested in performing inference on the dynamically evolving response of a structure when it is subjected to a stochastic excitation, e.g., an earthquake, based on uncertain information, e.g., sensor measurements.

KF and KS are algorithms for linear Gaussian models, which can be seen as special cases of the Dynamic Bayesian Network (DBN). The DBN is a probabilistic framework that models the evolution of a system or process over time. It consists of a sequence of connected Bayesian Networks (BNs), each representing the system at a time slice t [5]. The evolution in time is represented by directed links between nodes of successive time-slice BNs that carry information on temporal dependencies of the respective processes. Inference on the DBN for linear Gaussian systems can be performed using the KF and KS.

2.1. Probabilistic frameworks for structural health monitoring

Applications of the KF in SHM can be found in [6–8]. In these works, the Extended KF is used for system identification of linear and nonlinear systems. Studies using Bayesian methods in SHM have focused on identifying modal parameters of a structure and performing damage detection. A Bayesian framework to obtain distributions of the modal parameters, including the most probable values of the parameters and their uncertainties, is proposed in [9]. Au et al. [10] and Katafygiotis and Yuen [11] used data from ambient vibrations for modal identification. A Bayesian approach is proposed in [12] to account for uncertainties in the structural system to determine the existence and location of damage. Vanik et al. [13] used the proposed approach to continually update the stiffness parameters of a structure with a high likelihood of reduction in stiffness at a particular location used as a proxy for damage at that location. Rather than damage detection, we are interested in performing inference on the state of a structural system as it is subjected to a specific stochastic hazard.

For the monitoring of structures during extreme events, SHM systems are proposed in [14,15]. These studies are focused on the hardware aspects of the system rather than on performing probabilistic analysis of the data collected using these systems. Wu and Beck [16] used a Bayesian framework and expanded their analysis to the monitoring of a system both before and after an earthquake, with pre-event prognosis and post-event diagnosis. The response of the structure during the seismic event, however, is not analyzed. In general, previous studies using Bayesian methods for SHM limit the use of the Bayes rule to the standard Bayesian updating of system parameters. In this paper, we present a probabilistic framework to estimate the evolution of the structural response to stochastic excitation based on sensor measurements, and show the methodology to be robust to system uncertainties in performing this inference.

3. Method

3.1. System formulation

In the following, a capital bold letter denotes a matrix, such as the mass matrix $\mathbf{M}$, a small bold letter denotes a vector, as in the vector of structural displacements relative to the ground $\mathbf{u}_s(t)$, and a small italic letter denotes a scalar quantity, such as the ground displacement $u_g(t)$. Displacement and acceleration are denoted u and a, respectively, while $\mathbf{z}(t)$ collects displacement and velocity values. Subscripts g and s indicate quantities for the ground and structure, respectively.

We model the dynamical system as a cascaded system of two sub-systems: a ground sub-system and a structural sub-system, as shown in Fig. 1.

The ground dynamical sub-system takes a modulated white-noise input $w(t)$, representing the motion at the bedrock, and outputs the acceleration $a_g(t)$ on the ground surface. The structural dynamical sub-system takes $a_g(t)$ as well as ambient noise as excitation and produces the structural response $\mathbf{u}_s(t)$, the vector of nodal displacements relative to the ground. Our interest lies not only in inferring the instantaneous values of $\mathbf{u}_s(t)$ and related responses, but also in their peak values over time. This study assumes linear structural behavior as well as Gaussianity of both the earthquake and ambient-vibration input excitations to allow the use of Gaussian models and the KF described in the following sections. The proposed method can be extended to analyze nonlinear structural behavior by relaxing the assumption of a linear Gaussian system. As such, the current study is appropriate for operating-basis seismic events.

3.1.1. Ground dynamical sub-system

The equation describing the motion on the ground surface relative to the bedrock is given by

$$\ddot{u}_g + 2\xi_g\omega_g\dot{u}_g + \omega_g^2 u_g = -w \tag{1}$$

where ω_g and ξ_g define the angular frequency and damping ratio of the ground filter and w denotes the modulated white-noise acceleration at the bedrock. Written in first-order form with $\mathbf{z}_g = \begin{bmatrix} u_g & \dot{u}_g \end{bmatrix}^{\mathrm{T}}$, (1) becomes

$$\dot{\mathbf{z}}_g = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & -2\xi_g\omega_g \end{bmatrix}\mathbf{z}_g + \begin{bmatrix} 0 \\ -1 \end{bmatrix} w \tag{2}$$

The total acceleration at the surface of the ground, a_g, is obtained as

$$a_g = \ddot{u}_g + w = [0 \quad 1]\dot{\mathbf{z}}_g + w = \begin{bmatrix} -\omega_g^2 & -2\xi_g\omega_g \end{bmatrix}\mathbf{z}_g \tag{3}$$

3.1.2. Structural dynamical sub-system

The equation of motion for a linear structure subjected to base motion is

$$\mathbf{M}\ddot{\mathbf{u}}_s + \mathbf{C}\dot{\mathbf{u}}_s + \mathbf{K}\mathbf{u}_s = -\mathbf{M}\mathbf{i}a_g + \mathbf{f} \tag{4}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the mass, damping, and stiffness matrices, respectively, $\mathbf{i}$ is the influence vector relating the degrees of freedom to a unit base motion, and $\mathbf{f}$ models a random external force vector representing the effect of ambient noise, adding uncertainty to the system response. In first-order form, using $\mathbf{z}_s^{\mathrm{T}} = [\mathbf{u}_s^{\mathrm{T}}\ \dot{\mathbf{u}}_s^{\mathrm{T}}]$, (4) becomes

$$\dot{\mathbf{z}}_s = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}\mathbf{z}_s + \begin{bmatrix} \mathbf{0} \\ -\mathbf{i} \end{bmatrix} a_g + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}\mathbf{f} \tag{5}$$

3.1.3. State-space representation

Combining (2), (3) and (5), we obtain a representation of the full dynamical system in first-order form

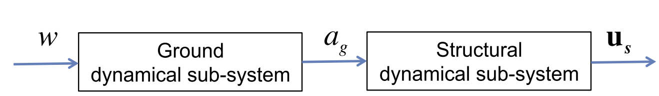

Fig. 1. Dynamical system model, consisting of ground and structural sub-systems.