Structural Safety 58 (2016) 60–68

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Random field-based modeling of size effect on the longitudinal tensile strength of clear timber

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ARTICLE INFO

Article history:
Received 6 November 2014
Received in revised form 8 September 2015
Accepted 14 September 2015
Available online 1 October 2015

Keywords:
Clear timber
Size effect
Longitudinal tensile strength
Random fields

ABSTRACT

In the current study, a random field-based size effect model has been proposed for the longitudinal tensile strength of clear timber. Since the failure mode is brittle, the problem is basically an extreme value problem of finding the distribution of minima of strength fields for specimens of different volumes. The stochastic response has been evaluated based on the Monte Carlo method along with the weakest link theory. Within the framework of the spectral representation method, the Weibull distribution has been considered as the marginal distribution to generate realizations of the strength as a 3D random field. The squared exponential autocorrelation function has been used for the description of spatial variability. The error resulting from this model, as compared to existing experimental data in the literature, is much lower than that of the classical Weibull law. The results show that when one of the specimen's dimensions decreases to less than 10 times the correlation length of the strength field, the size effect starts to deviate from the classical size effect law. Moreover, a simple analytical approximation, which includes the correlation length as length scale, has been presented that facilitates the application of the proposed model.

1. Introduction

From a structural point of view, wood/timber can be considered as a natural unidirectional fiber composite with highly anisotropic properties. For specific species, geographical location, and local growth conditions, the material properties depend on factors such as age, structural imperfections, location of timber within the tree, and load history and are therefore inherently highly variable. One of the consequences of this variability is the phenomenon known as statistical size effect. When the failure mode is brittle, the mean strength of a specimen with a larger volume is lower than that of a smaller one, which is also the case for any other level of cumulative probability, and this difference increases with the level of variability in material properties. This is normally attributed to the higher probability of critical defects occurring in a larger volume.

The classical Weibull size effect law (CWSEL) [1] is the most common model used in the literature [2] for the description of statistical size effects on timber strength in its brittle failure modes. According to this model, a structural member fails when the stress level reaches the strength at a single material point.

The size effect on timber strength can be treated as a volume effect, for example see [3–6] where such an assumption was adopted for the failure analysis of adhesively-bonded, welded and dovetail timber joints and [7–9] in which the elastoplastic behavior of strand-based wood composites and laminated veneer was studied. The Weibull law was applied only in tensile mode, while plastic behavior was considered only in compressive mode. Alternatively, the size effect can be split into length and cross-sectional effects – see e.g. [10,11] – on the strength prediction of clear timber under bending and [12] in the case of structural lumber.

Nevertheless, research efforts for the quantification of size effect on the strength of clear wood, by conducting pure tensile tests on specimens with different volumes, are very limited in the literature. Dill-Langer et al. [13] conducted longitudinal tensile experiments on specimens made of spruce wood. Two groups of specimens with different volumes were tested and a fiber bundle model was introduced in order to simulate the macroscopic behavior in terms of microscopic damage. Zhu et al. [14] introduced a length-effect parameter to quantify the size effect due to the length change on the longitudinal tensile strength of Japanese larch wood.

In current practice using the CWSEL, a Weibull distribution is fitted to data obtained from experiments on specimens with standardized dimensions. Then, the CWSEL is used to predict the strength of pieces of timber either with higher volumes [2,10,11],

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http://dx.doi.org/10.1016/j.strusafe.2015.09.002
0167-4730/

like timber beams, or lower volumes [4,7], which are usually small elements considered in finite element analyses of timber structures. However, using experimental data from the literature [13,14], it is shown in the current study that this procedure can result in relative errors in the prediction of the size effect on timber strength as high as 400%. This is attributed to the fact that the spatial correlation in the strength field is neglected in the CWSEL. It is noted that the inaccuracy of the CWSEL, when small volumes of materials are concerned, is already known; however, the issue has been overlooked in the case of timber. Nevertheless, it is important, particularly in the FE content, to extend size effect predictions to small volumes.

As an alternative to the CWSEL, timber strength can be modeled as a random field, considering, in addition to strength variation between different specimens as in conventional statistical analyses, the spatial variation within each specimen in a random manner. There are several methods to generate realizations of a random field in each specimen of which the spectral representation method is the most accurate and efficient [15]. A realization is a randomly generated sample of a random field and describes the spatial variability of the field for that sample in the physical domain considered, e.g. the volume of a specimen. Realizations of the strength field for specimens with different volumes can be used for investigating the size effect. Recently, Arwade et al. [16,17] used the random field approach to characterize the lengthwise spatial variability in the elastic modulus and compressive strength of parallel strand lumbers as 1D random fields. However, there is no study in the literature that has been devoted to the size effect on the longitudinal tensile strength of clear timber, and the spatial correlation in the strength field, i.e. the way in which the strength value at one point influences the strength at the surrounding points, has usually been neglected.

Studying the size effect on the longitudinal tensile strength of clear timber using random fields, is a problem of finding the extreme value statistics. This is because the failure mode is brittle, and to investigate the size effect, the distributions of minima of strength fields for specimens of different volumes have to be obtained. The extreme value problem was studied thoroughly by Gumbel in 1958 [18]. Concerning random fields, lots of research on this problem has been conducted in different scientific domains, e.g., by mathematicians [19], and in hydrology [20], climate [21], and cosmology [22]. Useful information on this topic can be found in [23]. Analytical solutions are only available for a few cases [24], and therefore, numerical methods such as simulations [25] or asymptotic approximation methods [24] are usually employed.

This study presents a model for the size effect on timber strength that takes into account the spatial variability in the strength field. The theory of random fields was used to model the random 3D spatial variability of the longitudinal strength. Using the spectral representation scheme, realizations of strength field in each specimen were generated considering the Weibull distribution and squared exponential function for statistical variability and spatial variability, respectively. The stochastic response was obtained via the Monte Carlo method along with the weakest link theory. This procedure was repeated for specimens with different volumes to estimate the effect of size on strength. The current results were compared to experimental data from the literature. An analytical expression able to efficiently approximate the numerical results of the random field modeling has also been introduced in this work.

2. Classical Weibull size effect law and its limitations

2.1. Weibull size effect theory

The CWSEL has been widely used to model the size effect on timber strength [2,4–12]. According to this model, the material is considered as a structure made up of linked elements, which fails with the first element failure. The mean strength of a specimen under uniaxial loading, $\bar{\sigma}(V)$, is related to its volume V as:

$$\bar{\sigma}(V) = \sigma_0 \Gamma(1 + 1/m)\left(\frac{V_0}{V}\right)^{1/m} \tag{1}$$

where σ_0 and m are the scale and shape parameters of the Weibull distribution, Γ is the Gamma function, and V_0 is a reference volume. Eq. (1) can be plotted as a straight line in a log–log scale. It can be shown that, at any given failure probability level, the strengths of two pieces, σ_1 and σ_2, with volumes V_1 and V_2 are related:

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{1/m} \tag{2}$$

based on the assumption of independent identically distributed random variables [26]. For a numerical representation of the method, the larger volume V_2 is divided into n segments with volumes equal to V_1 and a random strength value from the Weibull distribution, corresponding to volume V_1, is assigned to each segment. The minimum value of the segment strengths is considered as the strength of V_2 based on the weakest link theory. This simulation is repeated as many times as necessary to collect sufficient data points for estimating the statistics of the strength distribution of V_2. When V_2/V_1 is not an integer, interpolation can be used. Finally, having obtained the strength distributions for both volumes, it is easy to show that their relationship follows Eq. (2). However, due to this independent spatial assignment of the strength to smaller segments in the larger volume, the CWSEL tends to overestimate the effect of size on strength [26]. In reality, there is always a spatial correlation in the strength field that can be considered by using the random field approach. Moreover, Eq. (2) implies that the CWSEL can be scaled arbitrarily; i.e., its form does not change even for very small volumes of materials. Therefore, for very small volumes ($V \to 0$) the scale parameter, or equivalently the mean value of the strength distribution, approaches infinity. However, this is not the case in reality, and there is an upper limit for strength as volume decreases.

2.2. Modeling size effect on longitudinal tensile strength of clear timber with CWSEL

In most works related to the size effect on the longitudinal tensile strength of clear wood, tensile tests parallel to the grain are only performed on specimens with a constant volume. This is because testing a set of specimens with a constant volume suffices to determine the unknown parameter of the CWSEL. Thus, by fitting the Weibull distribution to the experimental data, the shape parameter is estimated, and the CWSEL is then used in the related application, such as estimation of a wooden joint capacity [4]. To the authors' knowledge, there are only two works [13,14] in which the effect of volume change on the longitudinal tensile strength of clear timber has been investigated experimentally. The specimen dimensions in these works and the corresponding Weibull parameters are given in Table 1. The dimensions correspond to the middle part of specimens with a constant cross-sectional area. In [13], longitudinal tensile tests were conducted on two sets of clear specimens made of spruce wood. The dimensions of the larger specimen are approximately three times those of the smaller specimen. In [14], however, only the length of the specimen has been changed. Nevertheless, in each of these works, the Weibull shape parameters obtained from fitting the experimental results of the smaller and larger specimens exhibit some differences. To examine the size effect within the Weibull theory framework, it is necessary to have a constant shape parameter; therefore, an average value was used in this study for examining the accuracy