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Effect of forcing frequency on nonlinear dynamic pulse buckling of imperfect rectangular plates with different boundary conditions

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ABSTRACT

The present study was aimed to investigate the influence of forcing frequency on nonlinear dynamic pulse buckling of imperfect rectangular plates with six different boundary conditions. The Galerkin's approximate method on the basis of polynomial and trigonometric mode shape functions is used to reduce the governing nonlinear partial differential equations to ordinary nonlinear differential equations. Moreover a numerical study of these governing equations is accomplished by Runge Kutta integration methods. The convergence of the polynomial and trigonometric mod shape functions are investigated to compute the dynamic response of plate. The effects of frequency of impulse loading and boundary conditions on the deflection histories of plate are studied. The dynamic response of plate subjected to impulsive loading with different forcing frequency is compared to results obtained by exponential impulsive loading. The results show that, by increasing the forcing frequency of impulsive loading, the maximum displacement of plate increases and converge with lower values to response of plate subjected to exponential impulse. Moreover, different boundary conditions and various pulse functions have significant influence on the dynamic response of the plate.

1. Introduction

With the advancement of industry, for the widespread applications of plates, the availability, durability, reliability, weight and strength, are known as the most important factors in optimum engineering design. In order to achieve an optimal design, mechanical properties and behavior of such structures in the presence of different loading conditions should be carefully investigated. Three types of loads, including lateral, axial and combined axial and lateral, can be applied to a plate. Due to the importance of buckling phenomena in the optimum design of plate, extensive buckling analysis has been carried out into two main categories, i.e. static buckling and dynamic buckling. Beams, columns, shells and plates are basic structural elements for Static buckling analysis and dynamic buckling analysis investigation [1–41]. Dynamic buckling response of plates subjected to impulsive loading has been the subject of considerable research interest.

Zizicas [27] was one of the first scientists investigated this subject. Neglecting the in-plane inertia effects in this research, the author studied the theoretical solution for simply supported plate under in-plane time-dependent load. Budiansky and Hutchinson [28,29] studied the simple imperfection-sensitive model to investigate the dynamical ability of structures. Danielson [30] developed the imperfection-sensitive model to include the effect of axial inertia. Applying a two-timing method the governing equations were solved to analysis. The nonlinear dynamic buckling behavior of imperfect rectangular plates under in-plane compressive step loads was studied theoretically by using the consistent perturbation technique. It was shown that, in pulse buckling, the in-plane inertia can be disregarded whereas in the latter, it plays an important role for non-slender plates [31]. Weller et al. [32] studied analytical investigations to calculate the dynamic load amplification factor (DLF) of metal beams and plates exposed to axial in-plane impact compressive loads by using the ADINA computer code.

The effect of anisotropic material properties on pulse buckling of imperfect rectangular plates under different sinusoidal pulse loading including quasi-static, dynamic and impulsive was investigated by Ari-Gur [33]. The results of this research showed that for a range of loading frequencies close to the fundamental frequency of the plate, the critical dynamic buckling loads were lower than the static ones.

Cui et al. [34] investigated the experimental dynamic buckling testes of rectangular plates with different boundary conditions under fluid-solid slamming. Using a stress failure criterion, the dynamic buckling of simply supported plates subjected to in-plane sinusoidal impact, rectangular and triangular pulses were studied. The effects of geometric dimensions, pulse functions, initial imperfections and limit stress of the material were perused in Ref. [35]. Cui et al. [36] carried out the numerical investigations of imperfect rectangular plate with different

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boundary conditions subjected to intermediate-velocity impact loads by using the computer code ABAQUS. The effect of initial imperfections, boundary conditions, dynamic load durations, and the hardening ratio of plate material on the dynamic buckling characteristics of the plates were also investigated. By employing the third-order shear deformation plate theory, Ma and Wang [37] solved the axisymmetric bending and buckling problems of functionally graded circular plates. It was shown that the first-order shear deformation plate theory results have enough accuracy and a much higher order and more complex plate theory was not necessary for such a kind of problem. Zenkour [38] showed that the classic plate theory, first-order shear deformation plate theory and third-order shear deformation plate theory results were confirmed by the second-order shear deformation plate theory obtained results. The dynamic buckling of thin-walled structures such as plates and beam-columns with open cross-section under compressive rectangular pulse loading was studied by Kubiak [39]. Meichael et al. [40] investigated a new refined hyperbolic shear deformation theory by using the Navier's solution technique for the buckling and free vibration analysis of FGM sandwich plates. The obtained results were validated by using classic plate theory, first-order, second-order and third-order shear deformation plate theory, parabolic shear deformation theory and 3D elasticity theory results. Kubiak [41] investigated the buckling and postbuckling response of thin plates and thin-walled structures with flat walls, under static and dynamic loading. The nonlinear dynamic pulse buckling of imperfect rectangular plate subjected to sinusoidal, exponential, damping and rectangular pulse functions with six different boundary conditions were investigated by Darvizeh et al. [42].

In this article, the effect of forcing frequency on nonlinear dynamic pulse buckling of imperfect rectangular plate with different boundary conditions is investigated. For this purpose, based on von Karman's nonlinear deformation theory, the nonlinear dynamic buckling governing equation of isotropic imperfect rectangular plate subjected to exponential-sinusoidal and exponential impulsive loading are derived through Hamilton's principle. The Galerkin method based on the polynomial and trigonometric Navier's double Fourier series is applied to transform the governing nonlinear partial differential equations to ordinary nonlinear differential equations and Runge Kutta method is used to obtain the displacement field. The convergence of the polynomial and trigonometric mod shape functions are investigated to compute the dynamic response of plate. The effects of frequency of impulse loading and six different boundary conditions on the deflection histories of plate are studied. The six different boundary conditions include SSSS, CSSS, CCSS, CSCS, CCCS and CCCC. In which S and C represent the simply supported and clamped boundary conditions for rectangular plates, respectively. The dynamic response of plate subjected to impulsive loading with different forcing frequency is compared to results obtained by exponential impulsive loading. The present paper extends the previous works [42] to investigate the effect of forcing frequency on nonlinear dynamic buckling of imperfect rectangular plates.

2. Governing equations

Consider an imperfect rectangular plate of thin uniform thickness h and length a, width b subjected to pulse loading as shown in Fig. 1. The displacements of an arbitrary point of coordinates (x, y) on the middle surface of the plate are denoted by u, v, w and initial imperfection, w_0 in the x, y and out-of-plane (z) directions, respectively. Considering initial imperfection, the Von Karman's nonlinear strain–displacement relationships can be written as

$$\varepsilon_x=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \tag{1}$$

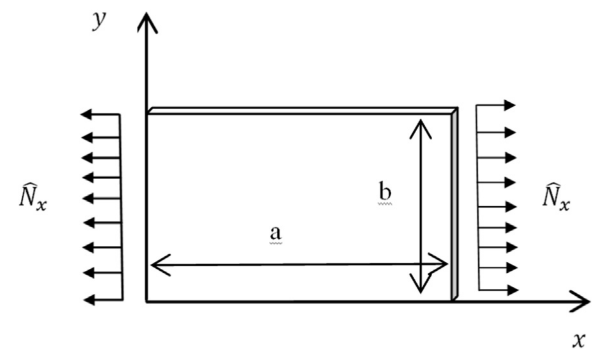

Fig. 1. Geometry and loading conditions of rectangular plate.

$$\varepsilon_y=\frac{\partial v}{\partial y}-z\frac{\partial^2 w}{\partial y^2}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2-\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \tag{2}$$

$$\varepsilon_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2z\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}-\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \tag{3}$$

The generalized Hooke's law constitutive equations in the case of in-plane stress-strain components are given by

$$\sigma_x=\frac{E}{1-\nu^2}(\varepsilon_x+\nu\varepsilon_y) \tag{4}$$

$$\sigma_y=\frac{E}{1-\nu^2}(\nu\varepsilon_x+\varepsilon_y) \tag{5}$$

$$\sigma_{xy}=\frac{E}{2(1+\nu)}\varepsilon_{xy} \tag{6}$$

In Eqs. (4)–(6), E is Young's modulus and ν is Poisson's ratio. The total potential energy of a rectangular plate on an elastic foundation is expressed as

$$\delta\left(\int_{t_0}^{t_1}(T-U+W)dt\right)=0 \tag{7}$$

where U represents the strain energy of the laminate, T the kinetic energy of the plate and W the potential energy.

$$U=\frac{1}{2}\iiint_V\left(\frac{1}{2E}\left(\sigma_x^2+\sigma_y^2-2\upsilon\sigma_x\sigma_y\right)+\frac{1}{2G}\sigma_{xy}^2\right)dV \tag{8}$$

$$T=\frac{1}{2}\rho\iiint_V\left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right\}dV \tag{9}$$

$$W=\frac{1}{2}\iint_A\left(N_x\left(\frac{\partial w}{\partial x}\right)^2+N_y\left(\frac{\partial w}{\partial y}\right)^2+2N_{xy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right)dxdy \tag{10}$$

Applying Hamilton principle along with varaitional method the dynamic buckling equation of motion for the lateral deflection can be derived.

$$\rho h\frac{\partial^2 w}{\partial t^2}+D\left(\nabla^4 w-\nabla^4 w_0\right)=N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+N_{xy}\frac{\partial^2 w}{\partial x\partial y} \tag{11}$$

where $D=\frac{Eh^3}{12\left(1-\nu^2\right)}$ is the flexural rigidity of the plate. Considering the general mid plane strain-displacement of imperfect plate and by applying differentiating ε_x twice respect to y, ε_y twice respect to x and ε_{xy} respect to x and y as following

$$\frac{\partial^2\varepsilon_x}{\partial y^2}=\frac{\partial^3 u}{\partial x\partial y^2}-z\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^3 w}{\partial x\partial y^2}\frac{\partial w}{\partial x}+\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial^2 w_0}{\partial x\partial y^2}\frac{\partial w_0}{\partial x}-\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2 w_0}{\partial x\partial y} \tag{12}$$