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Optimal design of uniaxially compressed perforated rectangular plate for maximum buckling load

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ABSTRACT

In this paper, the problem of linear elastic buckling of a simply supported rectangular plate, with a single circular cutout, subjected to uniform uniaxial edge compression is considered. The design objective is the maximization of the buckling load by determining the optimal location of the cutout. To accomplish that, a MATLAB routine coupled with finite element computation in ANSYS is employed. The study shows that the position of the center of the circular hole for the maximum buckling load remains always on the longitudinal centerline of the plate. However, the optimal center positions along this longitudinal axis for a given cutout size have great dependence on the aspect ratio of the plates. These optimal positions are reported for various values of plate aspect ratios and cutout sizes. These results are important from a design perspective and will be useful for design purposes in the future.

1. Introduction

This paper is motivated by the following design optimization problem. Consider a perforated rectangular plate and its stability under uniaxial edge compression. What would be the best possible location of the perforation for which the critical buckling load is maximum?

More precisely, we consider linear elastic buckling (or eigenvalue buckling) of a homogeneous isotropic simply supported thin rectangular plate, containing a circular cutout, subjected to uniform uniaxial edge compression. Our aim is to find the optimal location of the cutout in order to maximize the critical buckling load.

Thin plate structures are very common in engineering applications, especially in civil, mechanical and aerospace industries. There is often a need to introduce a cutout in such plate elements to provide access for the purpose of service, inspection and maintenance. The presence of these openings changes the stress distribution within the plate element and thereby results in variations in the buckling characteristics. Therefore, the performance of a plate structure containing hole is greatly influenced by its shape, size and location.

There are number of studies reported in the literature that discuss the effect of perforation parameters on the buckling behavior of a perforated plate. For example, Brown and co-authors [1–5] used the direct matrix method to determine the buckling load and found the effects of perforation size and position on the buckling load of the plate subjected to various combinations of in-plane loading. Shanmugam et al. [6] used finite element analysis to propose a design formula to predict the ultimate load capacity of perforated plates with different boundary conditions and subjected to both uniaxial and biaxial compressive load. El-Sawy and co-authors [7–9] used finite element analysis in ANSYS to study the effect of circular and rectangular cutouts on the linear and elasto-plastic buckling behavior of a simply supported plate subjected to both uniaxial and biaxial loading. Komur and Sonmez [10] studied the effect of the location of a circular cutout within a perforated rectangular plate under linearly varying in-plane edge load. Maiorana et al. [11] investigated the effect of circular and rectangular cut-outs on the buckling behavior of simply supported rectangular plates subjected to both in-plane compression and bending moment to propose the best position and orientation of these cutouts. They considered two orientations (horizontal and vertical) of the rectangular cutout in their study. Moen and Schafer [12] developed approximate expressions for predicting the elastic buckling stress of plates with single or multiple holes under compressive or bending loads. Using series of finite element analyses in ANSYS, Cheng and Zhao [13] investigated the buckling characteristics of strengthened perforated plates using various types of stiffeners. Komur [14] also used finite element computations in ANSYS to investigate the buckling behavior of simply supported uniaxially compressed rectangular plates having cutouts of elliptical shapes. There are few very recent papers [15–17] that also discuss the effect of cutouts on the elastic buckling behavior of plates that are subjected to various types of loading and displacement boundary conditions.

A common observation in all these above studies is that the cutout position within the plate has been chosen in the first step

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and then the effect of the same on the buckling load has been found. However, no work has been reported in the literature that employs an iterative approach of selecting the cutout position using an optimization algorithm in order to obtain the optimal cutout location. The objective of this paper is, therefore, to employ an optimization routine for obtaining the optimal locations of the cutout and report the optimal results so that some of these rigorous mathematical computations can be avoided later in the design processes.

2. Scope of the study

In this study, the general purpose commercial finite element software ANSYS is used for computing the critical buckling load for the simply supported rectangular plates with length a, width b, thickness h, containing a single circular cutout of radius r located at an arbitrary position (x_c, y_c) from the center of the plate. See Fig. 1. The plate is subjected to uniform uniaxial compressive in-plane loads (N_0).

Eigenvalue buckling analyses have been carried out for several cases after varying a number of parameters such as size of the cutout, plate aspect ratio, and the thickness of the plate. Cutout radius between 0.1b and 0.2b is considered. In addition, a minimum margin of 0.05b between the edges of the cutout and the plate is assumed as a design limitation. For all these cases the finite element analyses in ANSYS are coupled with an optimization routine in MATLAB and the optimal locations for each cases are obtained.

3. Method of study

3.1. Finite element computation

Eigenvalue buckling analyses in ANSYS [18] have been carried to obtain the critical buckling loads of the perforated plate. The four noded SHELL181 element with six degrees of freedom at each node is used to model the plate as it was satisfactorily used in similar studies in the literature [7,8,13,14]. This element considers the transverse shear deformation [18,19] and is suitable for analyzing thin to moderately thick shell structures. We carry out a verification study of this element and found satisfactory results when compared against some known analytical results (details not reported here). For generating the mesh, automatic meshing within ANSYS is used and the default element size all over the plate is taken as $b/40$. Along the hole perimeter a relatively finer mesh size is used. These mesh sizes are chosen based on convergence of the buckling results. A typical finite element mesh is shown in Fig. 2.

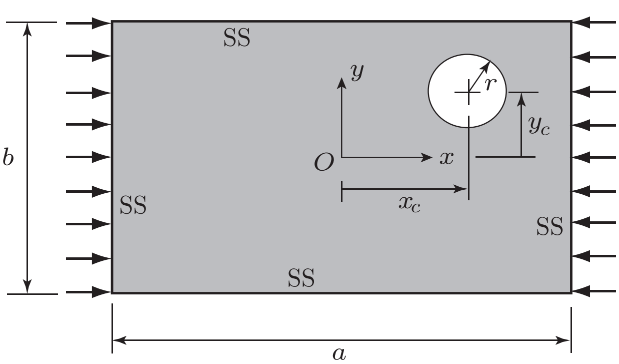

Fig. 1. Schematic of the uniaxially compressed rectangular plate containing a circular hole and having simply supported (SS) boundary conditions at all four edges.

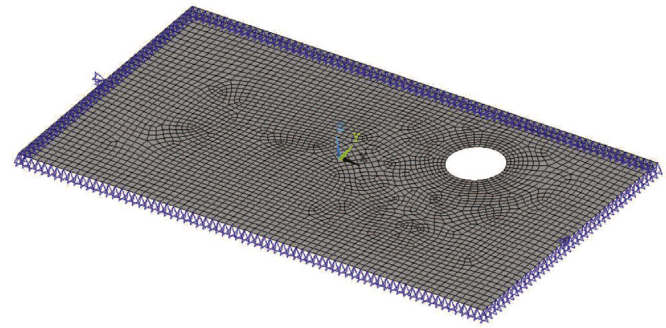

Fig. 2. A typical finite element mesh used for the buckling analysis of the perforated plate (a=1.8 m, b=1.0 m, r=0.1 m, x_c = 0.4 m, y_c = 0.2 m) with simply supported boundary conditions at all four edges. Here, U_z is restrained all along the edges. Additionally, U_x and U_y for one node in the $x = -a/2$ edge and U_y for another node in the x=a/2 edge are restrained to prevent the rigid-body motion of the plate.

The nodes along the edges of the plate are restrained appropriately to apply the simply supported boundary conditions. The displacements along z-direction (normal to the plane of the plate) are restrained. In addition, couple of nodes at the edges are restrained in appropriate directions such that only the rigid-body-displacement of the plate is restricted (see Fig. 2).

Young's modulus (E) and Poisson's ratio (ν) for the plate models used throughout this paper are taken as 210 GPa and 0.3 respectively. The eigenvalue buckling analyses of these plate models are carried out with an applied load of N_0 per unit width along the $x = \pm a/2$ edges of the plate. We note that the results obtained from the linear buckling analysis in ANSYS are the buckling load factors (say BF) that to be multiplied to the applied loads to reach the instability of the plate structure. Therefore, the total critical buckling load will be calculated as $N_{0cr} = BF \times N_0$. In this study, we are interested in the first instability of the plate and compute only the first buckling load factor.

3.2. Buckling loads vs cutout positions

As mentioned earlier, our aim in this study is to obtain the optimal location of the cutout position for the maximum buckling load. To motivate our study we first carry out number of buckling calculations of a simply supported perforated plate after varying the cutout location all along the plate. With these results, a surface plot of the critical buckling loads against the cutout locations is generated. See Fig. 3. Here, the buckling loads are normalized against the corresponding analytical buckling load ($\overline{N}_{0cr}$) of the *unperforated* plate. The analytical formula for the buckling of an unperforated plate, with all four edges simply supported, is given by [20]

$$\overline{N}_{0cr} = \frac{K\pi^2 E h^3}{12(1-\nu^2)b^2}, \tag{1}$$

where $\overline{N}_{0cr}$ is the critical buckling load per unit width and K is a constant that depends on the aspect ratio of the plate. We denote this *normalized buckling load factor* as

$$\text{NBF} = \frac{N_{0cr}}{\overline{N}_{0cr}}. \tag{2}$$

Fig. 3 shows that for a given plate aspect ratio (AR=a/b) the critical buckling load depends greatly on the position of the cutout and there indeed exists one optimal cutout location. And this optimal location varies with the aspect ratio of the plate. For example, we see that when AR=1 the optimal location is near the $x = \pm a/2$ edges whereas for AR=2 it is at the center of the plate. See Fig. 3(a) and (b). We also see from Fig. 3(c) that for AR=3 the