Thin-Walled Structures 96 (2015) 64–74

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Strength and stiffness requirements for intermediate ring stiffeners on discretely supported cylindrical shells

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ARTICLE INFO

Article history:
Received 19 June 2015
Received in revised form
28 July 2015
Accepted 3 August 2015

Keywords:
Cylindrical shells
Closed form solutions
Supports
Stiffening
Silos
Tanks

ABSTRACT

Silos in the form of a cylindrical metal shell are often supported on a ring beam which rests on discrete column supports. This support condition produces a circumferential non-uniformity in the axial membrane stresses in the silo shell. One way of reducing the non-uniformity of these stresses is to use a very stiff ring beam which partially or fully redistributes the stresses from the local support into uniform stresses in the shell. A better alternative is to use a combination of a flexible ring beam and an intermediate ring stiffener. Recent research by the authors has identified the ideal location of the intermediate ring stiffener to provide circumferentially uniform axial membrane stresses above the stiffener. To be fully effective, this intermediate ring should locally prevent both radial and circumferential displacements in the shell. This paper explores the strength and stiffness requirements for this intermediate ring stiffener. Pursuant to this goal, the cylindrical shell below the intermediate ring stiffener is analysed using the membrane theory of shells and the reactions produced by the stiffener on the shell are identified. These reactions are then applied to the intermediate ring stiffener. Vlasov's curved beam theory is used to derive closed form expressions for the variation of the stress resultants around the circumference to obtain a strength design criterion for the stiffener. A stiffness criterion is then developed by considering the ratio of the circumferential stiffness of the cylindrical shell to that of the intermediate ring stiffener. The circumferential displacements of the ring and the shell are found for the loading condition previously obtained to determine the required strength. A simple algebraic expression is developed for this intermediate ring stiffness criterion. These analytical studies are then compared with complementary finite element analyses that are used to identify a suitable value for the intermediate ring stiffness ratio for practical design.

1. Introduction

Silos in the form of cylindrical metal shells are usually discharged by gravity. A hopper is needed at the base of the cylindrical shell with an access space beneath it to permit discharge into transportation systems. The access space requirements increase when materials are directly discharged into trains, trucks, or conveying systems. As shown in Fig. 1, most silos are supported on columns at equal circumferential intervals to provide this space. There are stringent limitations on the number of column supports that can be used because the presence of a large number of columns does not allow for easy access by transportation systems.

Several different support arrangements may be chosen depending on the size of the structure [1]. For small silos, columns terminating at a ring, engaged columns or bracket supports may be suitable. Medium and large silos require either columns extending to the eaves (vertical stiffeners) or heavy ring beams. This paper addresses the most demanding situation of the large silo where a sizable ring beam is normally used, resting on discrete supports beneath the cylindrical shell, as shown in Fig. 1. The function of the ring beam is twofold. First, the ring beam is required to carry circumferential forces to maintain equilibrium at the transition junction between the cylinder and hopper [1]. Second, the ring beam plays an important role in redistributing the majority of the discrete forces from the column supports into a more uniform stress state in the cylindrical wall [1].

Previous studies of discretely supported cylinders [2–11] and those on ring beams above columns [12,13] have shown the great complexity of the behaviour. Since the design of the cylindrical shell is governed by considerations of buckling under axial compression, a much thinner wall can be provided if the axial membrane stress distribution is circumferentially uniform. Classical design treatments [14–17] adopted this assumption so that the criterion for buckling under axial compression above the ring is

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http://dx.doi.org/10.1016/j.tws.2015.08.004
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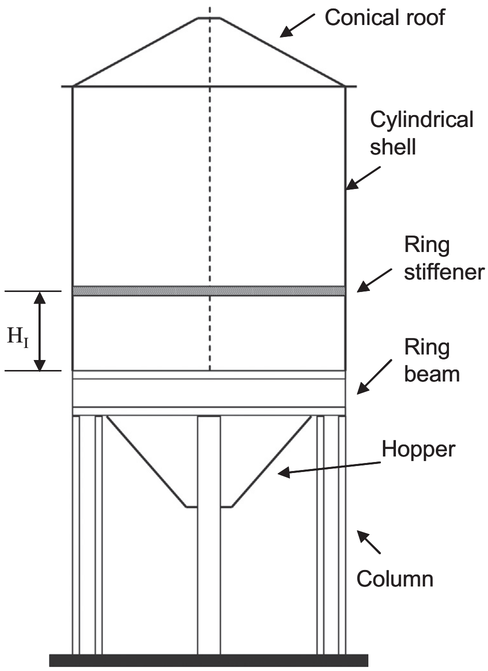

Fig. 1. Typical circular planform silo.

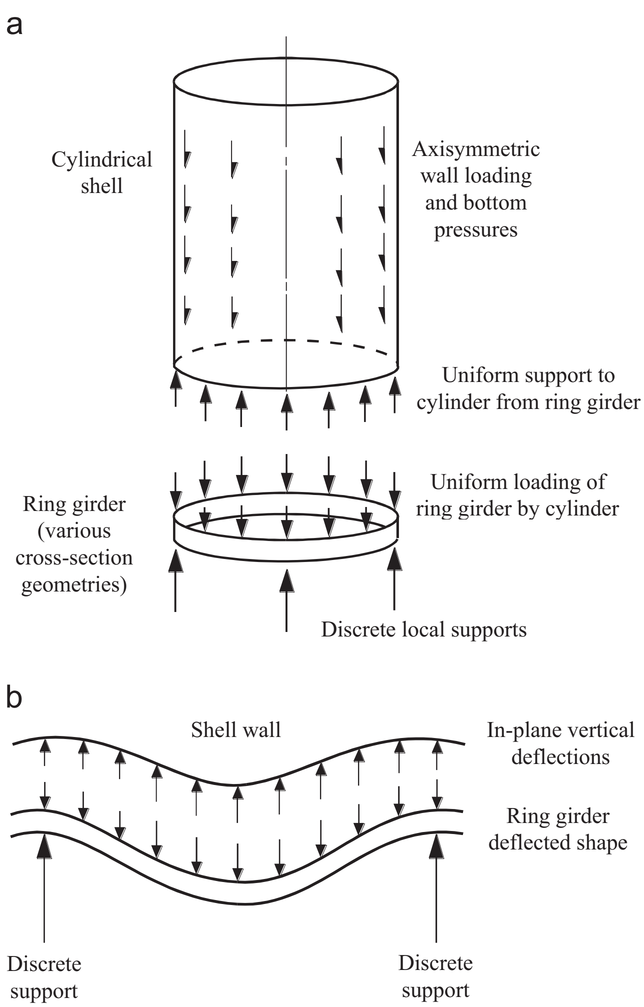

Fig. 2. Axial deformation compatibility between ring girder and shell (after [1]) (a) Traditional design model for column-supported silos. (b) Deformation requirement on cylinder imposed by compatibility with beam deformation.

that for uniform compression. But the underlying assumption can only be valid if the ring beam properly fulfils its critical function in redistributing the discrete support loads into a relatively uniform state of stress. The extent to which this redistribution of the support forces can be achieved is directly related to the stiffness of the ring beam relative to the stiffness of the cylindrical shell (Fig. 2). Since the cylindrical shell is very stiff in its own plane, the ring beam that is subject to flexure and twisting must be remarkably stiff to be stiffer than the shell. An approximate criterion to determine the appropriate ring beam stiffness was first identified by Rotter [13] and was further developed and verified by Topkaya and Rotter [18]. The criterion developed by these authors is very demanding and usually leads to very big ring beams for typical geometries.

One alternative method of achieving uniform axial membrane stresses is to use an intermediate ring stiffener as shown in Fig. 1. Greiner [19], Öry et al. [20] and Öry and Reimerdes [21] showed that an intermediate ring stiffener can be very effective in reducing the circumferential non-uniformity of axial stresses in the shell. Studies conducted by these researchers identified the variation of the axial membrane stress distributions up the height of the shell. It was shown that an intermediate ring stiffener can cause a dramatic decrease in the peak axial membrane stress, producing a more uniform stress state above the intermediate ring. Recently Topkaya and Rotter [22] showed that there is an ideal location for an intermediate ring stiffener, such that the axial membrane stress above this ring is circumferentially completely uniform. The ideal location is identified by the height H_I above the ring beam, defined as the vertical distance between the top of the ring beam and the centre of the intermediate ring stiffener as shown in Fig. 1. This was determined analytically and is expressed in terms of basic geometric variables. In passing, it should be noted that if a vertical stiffener is used between the main ring beam and the intermediate ring stiffener, a concentration of high axial stress is retained into the shell above the intermediate ring, so this arrangement is less useful.

The intermediate ring stiffener is expected to have sufficient strength and stiffness to fulfil its function properly. The key requirement for this intermediate ring stiffener is to prevent or significantly control the circumferential displacements of the cylindrical shell at that level. If the ring stiffener has inadequate stiffness then the circumferential uniformity of the axial stresses above it is not achieved. Furthermore, there is an interaction between the cylindrical shell and the ring stiffener which causes stress resultants to develop in the ring. These stress resultants could potentially cause failure of the ring stiffener either by yielding or by instability.

This paper presents a study that addresses the chief issues concerning the dimensioning of intermediate ring stiffeners. The general shell and ring combination is studied using the membrane theory of shells to revisit the derivation of the location of the ideal height and to identify the membrane shear forces induced in the shell by the ring. These forces are then considered as loads applied to the intermediate ring stiffener. Vlasov's curved beam theory is used to derive closed form expressions for the variation of the stress resultants around the circumference to obtain a suitable strength design criterion for the stiffener. A relative stiffness criterion for the ring is then devised by considering the ratio of the circumferential stiffness of the cylindrical shell to that of the intermediate ring. Using the same loads on the ring as for the strength determination, the circumferential displacements of the ring and the shell are derived. A simple algebraic expression is then developed for this relative stiffness. These analytical studies are then verified using a wide range of finite element analyses to identify a suitable value for use in practical design.