Tunnelling and Underground Space Technology 58 (2016) 209–215

Contents lists available at ScienceDirect

Tunnelling and Underground Space Technology

journal homepage: www.elsevier.com/locate/tust

Analytical solution for long lined tunnels subjected to travelling loads

Haitao Yu [a], Chuang Cai [b], Xiaofei Guan [c], Yong Yuan [d,*]

[a] Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, Shanghai 200092, China
[b] Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China
[c] Department of Mathematics, Tongji University, Shanghai 200092, China
[d] State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China

ARTICLE INFO

Article history:
Received 2 September 2015
Received in revised form 7 April 2016
Accepted 25 May 2016
Available online 2 June 2016

Keywords:
Long lined tunnel
Travelling loads
Dynamic response
Integration transform
Convolution theorem

ABSTRACT

An analytical solution is derived for dynamic response of long lined tunnels subjected to travelling loads. For the derivation, the long lined tunnel is assumed to be infinitely long with a uniform cross-section resting on a viscoelastic foundation. Fourier and Laplace transforms are utilized to simplify the governing equation of the tunnel to an algebraic equation, so that the solution can be conveniently obtained in the frequency domain. The convolution theorem is employed to convert the solution into the time domain. Final solutions of tunnel responses investigated are deflection, velocity, acceleration, bending moment, and shear force. The proposed solution is verified by providing comparisons between its results and those from the Finite Element program ABAQUS. Further parametric analysis, such as the influence of wave velocity and frequency on dynamic responses of the tunnel is presented with the analytical solution. These relationships can be an effective tool for practitioners.

1. Introduction

Investigation on dynamic response of tunnels subjected to seismic waves is a classical problem in anti-seismic research of underground structures, and has significance in seismic design of tunnel engineering. During the propagation of seismic wave, the time when the wave arrives varies with different sites. It has been noted that appropriate considerations should be given to travelling waves (Hwang and Lysmer, 1981), especially for extended or embedded structures with a large span or size such as tunnels, because spatial variation of earthquake motion has dramatic effects, which was already noted by a number of researchers (Park et al., 2009; Yu et al., 2013b; Li and Song, 2015). Consequently, the influence of the wave-passage effect on the safety of tunnel structures should be quantitatively evaluated and highly considered in seismic design.

Current research on tunnel response induced by travelling loads is limited to numerical approaches, such as the Finite Difference Method, Finite Element Method, and Boundary Element Method. Several studies have been performed on this issue. Stamos and Beskos (1995) proposed a frequency domain boundary element method to investigate the dynamic response of three-dimensional underground structures subjected to dynamic disturbances with a harmonic or a transient time variation. Later, Stamos and Beskos (1996) employed a special direct boundary element method in the frequency domain for both the tunnel and the soil, assuming the long lined tunnel to be infinitely long with a uniform cross-section buried into an elastic or viscoelastic half-space to body and surface harmonic seismic waves, which effectively reduces the three-dimensional problem to a two-dimensional one. Park et al. (2009) performed a series of pseudo-static three-dimensional finite element analysis to evaluate the longitudinal tunnel response under spatially varying ground motion. Yu et al. (2013a, 2013b) proposed a multiscale method to simulate dynamic responses of long tunnels subjected to uniform and non-uniform seismic loadings, which involves the concurrent discretization of the entire domain with both coarse- and fine-scale finite element meshes. Li and Song (2015) developed a 3-D finite element model in time domain to provide a feasible computational modeling technique for the longitudinal seismic response of tunnels under an asynchronous earthquake wave input. In these analyses, one has to consider both time and spatial increments in the numerical approaches, and thus these methods are time-consuming and the numerical accuracy is correlated to the integration algorithm. Computational efficiency can be improved if analytical solutions are available.

Clearly, analytical formulations are limited due to the assumptions that need to be made to reach the solution. In most cases, the

* Corresponding author at: Department of Geotechnical Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China.
E-mail addresses: yuhaitao@tongji.edu.cn (H. Yu), yuany@tongji.edu.cn (Y. Yuan).

http://dx.doi.org/10.1016/j.tust.2016.05.008
0886-7798/

design will require a numerical method that does not have the shortcomings of the analytical solutions, as it can consider the construction process, non-linear behavior, etc. Closed-form solutions however are invaluable to obtain a better understanding of the interplay that exists between dynamic loads, viscoelastic foundation and tunnel structure, to identify what are the most critical parameters for the problem, and to provide first estimates or even a preliminary design. An added advantage is that they can be used with very little cost to conduct sensitivity analysis and, most importantly, to provide benchmark values to check the results of the more complex numerical models.

St. John and Zahrah (1987) used Newmark's approach to develop closed-form solutions for free-field axial and curvature strains due to compression, shear and Rayleigh waves. Based on this, they used the pseudo-static approach, i.e. the free field deformation approach, to estimate the strains and curvature of the tunnel subjected to a harmonic motion propagating at an angle to the tunnel axis. However, the free field approach ignores the inertia forces and the interaction between the tunnel and the surrounding ground, and thus may overestimate or underestimate structure deformations depending on the rigidity of the structure relative to the ground, which was already noted by a number of researchers (Hashash et al., 2001; Bobet, 2003; Huo et al., 2006).

This paper focuses on the analytical solution for the dynamic response of long lined tunnels subjected to travelling loads, and taking into account both the inertia forces and the interaction between the soil and the structure. Several assumptions are made for the problem: the long lined tunnel is assumed to be infinitely long with a uniform cross-section and to behave as linear elastic; the surrounding soil medium is assumed to be isotropic and homogeneous and to behave as viscoelastic; the travelling loads are assumed to be plane harmonic loads and propagate parallel to tunnel axis. To obtain the analytical solution of the problem, the Fourier transform is used to simplify the governing equation of the tunnel in space domain, whereas the Laplace transform is employed to reduce the equation in time domain. The governing equation of the tunnel based on the integration transform, therefore, is changed to an algebraic equation so that the solution can be conveniently given in the frequency domain. Finally, the convolution theorem is employed to convert the solution into the time domain. Various examples involving travelling loads and uniform loads are presented to illustrate the solutions, and the results are compared against those of the finite element method in order to assess its accuracy. Parametric analyses are also performed to investigate the influence of the wave-passage effect on dynamic responses of the tunnel structure. These relationships can be conveniently used to obtain the tunnel response and can be an effective tool for practitioners.

2. Governing equation

Consider the idealised model drawn out for a practical engineering problem of an infinitely long lined tunnel of uniform cross-section resting on a viscoelastic foundation and subjected to plane harmonic travelling loads along the longitudinal direction of the tunnel. Fig. 1 depicts the coordinate system and significant dimensions associated with an infinite long lined tunnel. The tunnel with constant stiffness EI and mass per unit length ρA is considered, where E = Young's modulus of elasticity; I = moment of inertia of the tunnel cross section; ρ = density of the tunnel liner; A = area of cross section of the tunnel. The tunnel is supported by a viscoelastic foundation with constant spring stiffness K and viscous damping C per unit length.

Define $y(x,t)$ as the vertical deflection of the tunnel and $F(x,t)$ as the plane harmonic travelling loads, in which the loads propagate parallel to the tunnel axis (see the x-axis in Fig. 1) and t is time. The wave-passage loads can be expressed as

$$F(x,t)=\begin{cases}0, & (x>Vt)\\ P\sin\left[2\pi\Omega\left(t-\frac{x}{V}\right)\right], & (x\leqslant Vt)\end{cases} \tag{1}$$

where V, Ω and P = wave velocity, frequency and amplitude of the loads, respectively.

The tunnel structure is assumed to behave linear elastically and deform only due to the normal travelling loadings perpendicular to the tunnel axis (deformations of the tunnel structure due to axial forces are neglected). This is a common assumption in structural mechanics. To simplify the derivation, the shear distortion of the tunnel cross section is neglected here, and thus the tunnel structure can be assumed to behave as an Euler-Bernoulli beam (Yu and Yuan, 2014). The governing equation of the tunnel with constant cross section resting on a viscoelastic foundation subjected to travelling loads is given by

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}+C\frac{\partial y(x,t)}{\partial t}+Ky(x,t)=F(x,t) \tag{2}$$

Assume that the tunnel is at rest prior to the travelling loads being applied. This means that the initial conditions of the tunnel displacement and velocity are zero, i.e.

$$y(x,t)|_{t=0}=0,\quad \frac{\partial y(x,t)}{\partial t}|_{t=0}=0 \tag{3}$$

For an infinitely long lined tunnel, the boundary conditions are

$$\lim_{x\to\pm\infty}\frac{\partial^n y(x,t)}{\partial x^n}=0\quad (n=0,1,2,3) \tag{4}$$

Eqs. (1)–(4) constitute the complete mathematical description of the problem. The dynamic response of the tunnel with respect to the applied travelling loads can be obtained by solving these linear partial-differential equations.

3. Integral representation of the solution

Eq. (2) is a linear partial-differential equation. Integral transformation will enable its conversion into an algebra equation.

Define the Fourier transform and its inversion (Eringen and Suhubi, 1975)

$$\varphi(u)=\mathbf{F}[f(x)]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x)e^{iux}dx \tag{5}$$

$$f(x)=\mathbf{F}^{-1}[\varphi(u)]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\varphi(u)e^{-iux}du \tag{6}$$

where $\mathbf{F}[\cdot]$ and $\mathbf{F}^{-1}[\cdot]$ = Fourier transform and its inversion, respectively.

Define the Laplace transform and its inversion (Morse and Feshbach, 1953)

$$\varphi(s)=\mathbf{L}[f(t)]=\int_0^{+\infty}f(t)e^{-st}dt,\quad t>0 \tag{7}$$

$$f(t)=\mathbf{L}^{-1}[\varphi(s)]=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\varphi(s)e^{st}ds,\quad t>0,\ s>0 \tag{8}$$

where $\mathbf{L}[\cdot]$ and $\mathbf{L}^{-1}[\cdot]$ = Laplace transform and its inversion, respectively; and γ = an arbitrary real-valued number such that the path integral in the complex-s plane for Eq. (8) lies in the right-hand side of all poles of $f(s)$.