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A Bayesian approach to modeling group and individual differences in multidimensional scaling

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HIGHLIGHTS

- We consider possible group and individual differences in MDS representations.
- We develop a novel Bayesian implementation of the K-INDSCAL model.
- We apply the model to three psychological data sets.
- The results demonstrate the different sorts of group-level and individual-level differences.

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ABSTRACT

Multidimensional scaling (MDS) models of mental representation assume stimuli are represented by points in a low-dimensional space, such that more similar stimuli are represented by points closer to each other. We consider possible individual differences in MDS representations, using the recently proposed K-INDSCAL model, which allows for both sub-groups of people with different representations, and individual differences in the attention people give to different stimulus dimensions. We develop a novel Bayesian implementation of the K-INDSCAL model, and demonstrate in a simulation study it is capable of inferring meaningful individual differences for the sorts of data sets typically available in psychology. We then apply the model to three existing data sets, involving the taste of colas, images of cats, and colors of different hues. Collectively, the results demonstrate the flexibility of the K-INDSCAL model in finding both group- and individual-level differences, and highlight the need for Bayesian methods to make these inferences.

1. Introduction

Multidimensional scaling (MDS) is a long-established and widely-used statistical method for finding a spatial representation of a set of objects, based on the similarities between those objects (Borg & Groenen, 2005; Cox & Cox, 1994; Schiffman, Reynolds, Young, & Carroll, 1981). MDS represents objects as points in a low-dimensional space, so that the similarity between each pair of objects corresponds to the distance between the points representing them, with more similar objects being nearer each other. Used as a statistical method, MDS has been applied throughout the natural and human sciences as a dimensionality-reduction method, providing useful representations and visualizations of the relationships between objects. In these sort of applications, the required similarities or proximities between each pair of objects are often found by automated methods, such as calculating the correlations or distances between vectors of information representing the objects. For example, the similarities of text documents can be derived from counts of the number of keywords they have in common, or the similarity between countries might be based on (standardized) differences in properties like their landmass, GDP, population, and so on.

The MDS model, however, has its origins in theories of human mental representation (Shepard, 1974). MDS representations have provided psychologically meaningful representations of many stimulus domains, especially low-level perceptual domains such as colors, phonemes, and simple perceptual forms (Shepard, 1980). Shepard (1987) developed a formal theory of the key cognitive process of stimulus generalization based on MDS representations, and MDS models are widely used to represent stimuli in cognitive models of identification, categorization, and learning (Nosofsky, 1992). Used as a psychological model of stimulus

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representation, the similarities or proximities between each pair of stimuli are usually based on human judgments of similarity. These behavioral judgments can take the form of ratings on a scale, or be derived from various decision-making tasks, such as triadic comparison (Romney, Brewer, & Batchelder, 1993), or identification or confusion tasks (Miller & Nicely, 1955).

When MDS is used as a statistical method of dimensionality reduction, there is little conceptual scope to consider individual differences, because the similarities are calculated from known underlying quantitative descriptions. When MDS is used as a psychological model of representations, however, the question of individual differences becomes important. Different people may represent the similarities between stimuli differently, and a good model of human mental representation should account for these differences. One well-established generalization of the MDS model that include individual differences is the INDSCAL model (Carroll & Chang, 1970; Takane, Young, & De Leeuw, 1977), which still assumes a single spatial representation, but allows individual differences by placing subject-specific weights on each dimension. Thus, differences in judged similarities between stimuli are explained by different people placing different emphasis on the dimensions. Another approach to allowing individual differences in MDS representations is provided by latent-mixture modeling (Lee, 2008; Winsberg & De Soete, 1993), in which there are multiple spatial representations, each corresponding to a subgroup of subjects. Recently, Bocci and Vichi (2011) proposed a generalization of the MDS called K-INDSCAL, which effectively combines the INDSCAL and latent-mixture models of individual differences. K-INDSCAL is a K latent-class mixture of INDSCAL, and so includes the possibility of different spatial representations for groups of subjects, as well as dimension weights for individual subjects within each group.

While the K-INDSCAL model is an important theoretical development, the implementation reported by Bocci and Vichi (2011) uses classical estimation methods based on least-squares optimization. For a model with the complexity and richness of K-INDSCAL, this is potentially limiting, because it does not allow for the measurement of uncertainty in inferences. Given the model allows both group-level and individual-level differences in stimulus representation, most data will be at least partly consistent with different representational possibilities, and it is important to understand what can and cannot be inferred about individual differences. The ability to handle uncertainty in a complete and coherent way is the cornerstone of the Bayesian approach to statistical inference (Jaynes, 2003; Lindley, 1972). There is a modest and growing literature on Bayesian methods for MDS modeling (Bakker & Poole, 2013; Lee, 2008; Oh & Raftery, 2001; Okada & Mayekawa, 2011; Okada & Shigemasu, 2010; Park, DeSarbo, & Liechty, 2008), but it has yet to be applied to the K-INDSCAL model. There is also a literature on using Bayesian methods to understand individual differences in cognitive processes related to similarity judgment, including category learning (Bartlema, Lee, Wetzels, & Vanpaemel, 2014), memory (Dennis, Lee, & Kinnell, 2008), and decision-making (Lee, 2015; Scheibehenne, Rieskamp, & Wagenmakers, 2013; van Ravenzwaaij, Dutilh, & Wagenmakers, 2011).

Accordingly, the goal of this paper is to develop, evaluate, and apply a Bayesian approach for the K-INDSCAL model, using modern computational Markov chain Monte Carlo (MCMC) methods. In the next section, the Bayesian K-INDSCAL model is formally defined, and we develop a two-step post-processing method that can be applied to MCMC samples taken from its posterior to deal with indeterminacy issues. We report the results of a small simulation study using artificially generated data to check that inferences are possible from the sorts of data typically available in psychology. We then present three applications of the proposed Bayesian method using previously considered psychological data. The three applications involve the taste of colas, images of cats, and colors of different hues, and collectively highlight the flexibility of the model to consider both group-level and individual-level differences in representation. Finally, we discuss the current contributions and future directions for the modeling approach.

2. Model and inference method

2.1. A Bayesian formulation of the K-INDSCAL model

Our proposed Bayesian method is an extension of previous work by Oh and Raftery (2001). These authors developed a Bayesian approach for applying a standard MDS model to data. We develop a Bayesian approach for the K-INDSCAL extension of the standard MDS model. For this model, the data take the form of three-way observed dissimilarities between pairs of I stimuli judged by H subjects. We denote the observed proximity matrix of subject h as $\boldsymbol{Y}_h$ $(h = 1, \ldots, H)$, where the (i, i')-th element, $y_{ii'h}$, represents the perceived dissimilarity between stimuli i and i' judged by the subject. The model assumes that the observed dissimilarity is the sum of the true distance and measurement error:

$$y_{ii'h} \sim N_{[0,\infty)}(d_{ii'h}, \sigma^2). \tag{1}$$

In this equation, $N_{[0,\infty)}(\cdot)$ represents truncated normal distribution, which is used because the observed dissimilarity does not take negative values (Oh & Raftery, 2001), $d_{ii'h}$ represents the distance between stimuli i and i' perceived by subject h who belongs to class k $(h = 1, \ldots, H,\ k = 1, \ldots, K, K < H)$. This membership is represented by an indicator variable $u_h = k$. The distance is given as the weighted Euclidean distance

$$d_{ii'h} = \sqrt{\sum_{j=1}^{J} w_{hj}(x_{ijk} - x_{i'jk})^2}. \tag{2}$$

The spatial representation matrix $\boldsymbol{X}_k = \{x_{ijk}\}$ $(k = 1, \ldots, K)$, which represents the coordinates of I stimuli in a J-dimensional space, is shared by all the subjects who belong to the same class k. The weight that subject h gives to dimension j, $\boldsymbol{W} = \{w_{hj}\}$, however, differs from one subject to another, representing individual differences within the class. The latent parameters $\boldsymbol{X}_k$, $\boldsymbol{W}$ and $\boldsymbol{u} = \{u_h\}$ are to be inferred from observed data using the model.

As required by the Bayesian approach, we place prior distributions over all of the parameters. For the prior on $\boldsymbol{X}_k$ we use the normal distribution

$$x_{ijk} \sim N(0, \tau_j^2). \tag{3}$$

For the prior on the variance parameters we use the standard assumption of an inverse gamma distribution,

$$\sigma^2 \sim IG(\alpha_\sigma, \beta_\sigma), \tag{4}$$

$$\tau_j^2 \sim IG(\alpha_\tau, \beta_\tau). \tag{5}$$

The hyper-parameter values, $\{\alpha_\sigma, \beta_\sigma, \alpha_\tau,$ and $\beta_\tau\}$ can be set to small values to represent little prior information. In all of the modeling we report, they were set to 0.001.

The individual weights $\boldsymbol{W} = \{w_{hj}\}$ are assumed to lie in the J-simplex for each subject, so that

$$w_{hj} \geq 0, \quad \sum_{j=1}^{J} w_{hj} = J. \tag{6}$$

For example, when $J = 2$, if the first subject considers the first dimension to be slightly more important, this may correspond to a