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What underlies successful word problem solving? A path analysis in sixth grade students

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ABSTRACT

Two component skills are thought to be necessary for successful word problem solving: (1) the production of visual-schematic representations and (2) the derivation of the correct relations between the solution-relevant elements from the text base. The first component skill is grounded in the visual–spatial domain, and presumed to be influenced by spatial ability, whereas the latter is seated in the linguistic–semantic domain, and presumed to be influenced by reading comprehension. These component skills as well as their underlying basic abilities are examined in 128 sixth grade students through path analysis. The results of the path analysis showed that both component skills and their underlying basic abilities explained 49% of students' word problem solving performance. Furthermore, spatial ability and reading comprehension both had a direct and an indirect relation (via the component skills) with word problem solving performance. These results contribute to the development of instruction methods that help students using these components while solving word problems.

1. Mathematical word problem solving

Mathematical word problem solving plays a prominent role in contemporary mathematics education (Rasmussen & King, 2000; Timmermans, Van Lieshout, & Verhoeven, 2007). The term *word problem* is used to refer to any math exercise where significant background information on the problem is presented as text rather than in mathematical notation. As word problems often involve a narrative of some sort, they are occasionally also referred to as *story problems* (Verschaffel, Greer, & De Corte, 2000). An example of a word problem is given below (taken from Hegarty & Kozhevnikov, 1999):

Example 1. At each of the two ends of a straight path, a man planted a tree and then, every 5 m along the path, he planted another tree. The length of the path is 15 m. How many trees were planted?

Students often experience difficulties in the understanding of the text of a word problem, rather than its solution (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Lewis & Mayer, 1987). Two component skills are thought to be necessary for successful word problem solving: (1) producing visual-schematic representations (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Montague & Applegate, 2000; Van Garderen & Montague, 2003) and (2) relational processing, that is deriving the correct relations between the solution-relevant elements from the text base (e.g., Hegarty, Mayer, & Monk, 1995; Kintsch, 1998; Van der Schoot, Bakker Arkema, Horsley, & Van Lieshout, 2009; Verschaffel, 1994; Verschaffel, De Corte, & Pauwels, 1992). These two component skills are presumed to explain unique variance in students' word problem solving performance and cover different processing domains (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van der Schoot et al., 2009). The production of visual-schematic representations is grounded in the visual–spatial domain (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Mayer, 1985; Van Garderen, 2006), whereas relational processing is seated in the linguistic–semantic domain (e.g., Pape, 2003; Thevenot, 2010; Van der Schoot et al., 2009). These component skills, as well as the basic abilities which underlie each of these skills, are described below.

1.1. Component skill in the visuo-spatial domain: The production of visual-schematic representations

Rather than the superficial selection of numbers and relational keywords from the word problem text (often resulting in the execution of the wrong arithmetic operations), good word problem solvers generally construct a visual representation of the problem to facilitate understanding (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Montague & Applegate, 2000; Van der Schoot et al., 2009). With this, the nature of these visual representations determines their effectiveness. According to Hegarty and

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Kozhevnikov (1999), two types of visual representations exist: *pictorial* and *visual-schematic* representations. Children who create pictorial representations tend to focus on the visual appearance of the given elements in the word problem. These representations consist of vivid and detailed visual images (Hegarty & Kozhevnikov, 1999; Presmeg, 1997). However, several studies have reported that the production of pictorial representations is negatively related to word problem solving performance (Ahmad, Tarmizi, & Nawawi, 2010; Hegarty & Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Mayer, 2002; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). An explanation for this finding is that children who make pictorial representations fail to form a coherent visualization of the described problem situation and base their representations solely on a specific element or sentence in the word problem text (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). Children who make visual-schematic representations do integrate the solution-relevant text elements into a coherent visualization of the word problem (e.g., Ahmad et al., 2010; Krawec, 2010; Van Garderen, 2006). This explains why, in contrast to the production of pictorial representations, the production of visual-schematic representations is found to be positively related to word problem solving performance (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006; Van Garderen & Montague, 2003).

1.1.1. Basic ability in the visuo-spatial domain: Spatial abilities

The production of visual-schematic representations is influenced by spatial ability. Children with good spatial skills have been found to be better able to make visual-schematic representations than children with poor spatial skills (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). Although there are many definitions of what spatial ability is, it is generally accepted to be related to skills involving the retrieval, retention and transformation of visual information in a spatial context (Velez, Silver, & Tremaine, 2005). Especially the involvement of a specific spatial factor – that is, *spatial visualization* – in making coherent visual-schematic representations has been made clear by several authors (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). Spatial visualization refers to the ability to mentally manipulate objects (i.e. mental rotation; Kaufmann, 2007; Voyer, Voyer, & Bryden, 1995). In the present study, spatial ability refers to spatial visualization as described above.

Besides the role of spatial ability in word problem solving via the production of visual-schematic representations, several authors also report a *direct* relation between spatial ability and word problem solving (Battista, 1990; Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007; Booth & Thomas, 1999; Edens & Potter, 2008; Geary, Saults, Liu, & Hoard, 2000; Hegarty & Kozhevnikov, 1999; Orde, 1997). Blatto-Vallee et al. (2007), for example, showed that spatial abilities explained almost 20% of unique variance in word problem solving performance. Casey and colleagues revealed that the direct role of spatial abilities in word problem solving lies in performing the actual mathematical operations and numerical reasoning (e.g., Casey, Nuttall, & Pezaris, 1997, 2001; Casey et al., 2008).

1.2. Component skill in the linguistic–semantic domain: Relational processing

Although the production of visual-schematic representations is a necessary condition for successful word problem solving, it is not always a sufficient condition (Kintsch, 1998; Pape, 2003; Van der Schoot et al., 2009), since children may be very well capable of forming a visual-*schematic* representation without being able to infer the *correct* relations between the solution-relevant elements from the word problem text (Coquin-Viennot & Moreau, 2003; Krawec, 2010; Thevenot, 2010). Relational processing in word problem solving can be effectively revealed in word problems in which the relational term maps onto non-obvious mathematical operations (De Corte, Verschaffel, & De Win, 1985; Kintsch, 1998; Thevenot, 2010; Thevenot & Oakhill, 2005, 2006; Van der Schoot et al., 2009). In word problems with an obvious mapping, it is sufficient to first select the numbers and relational terms from the text and then to directly translate these into a set of computations (Hegarty et al., 1995; Pape, 2003; Van der Schoot et al., 2009). However, in non-obvious word problems, other text elements are necessary for the construction of an effective mental model of the word problem including the appropriate relations between the key variables (De Corte et al., 1985; Thevenot, 2010; Thevenot & Oakhill, 2005, 2006; Van der Schoot et al., 2009). Consider, for example, the following word problem in which the relation term 'more than' primes an inappropriate mathematical operation:

Example 2. At the grocery store, a bottle of olive oil costs 7 euro.

That is 2 euro 'more than' at the supermarket.

If you need to buy 7 bottles of olive oil, how much will it cost at the supermarket?

In this so-called inconsistent word problem (Hegarty, Mayer, & Green, 1992; Hegarty et al., 1995; Kintsch, 1998; Van der Schoot et al., 2009), the crucial arithmetic operation (i.e. 7-2) cannot be simply derived from the relational keyword ('more than'). Rather than making use of a superficial, direct-retrieval strategy (Giroux & Ste-Marie, 2001; Hegarty et al., 1995; Thevenot, 2010; Verschaffel, 1994; Verschaffel et al., 1992), problem solvers have to appeal to a problem-model strategy in which they translate the problem statement into a qualitative mental model of the base type of situation (in this case: a subtraction situation) that is hidden in the problem. Here, this translation requires the identification of the pronominal reference 'that is' as the indicator of the relation between the value of the first variable ('the price of a bottle of olive oil at the grocery store') and the second ('the price of a bottle of olive oil at the supermarket'). On the basis of the constructed mental model, problem solvers are then able to plan and execute the required arithmetic operations. Hence, inconsistent word problems are suitable to measure relational processing.

1.2.1. Basic ability in the linguistic–semantic domain: Reading comprehension

Previous studies have shown that the role of relational processing in word problem solving is influenced by a child's reading comprehension abilities (e.g., Lee, Ng, Ng, & Lim, 2004; Van der Schoot et al., 2009). For example, Lewis and Mayer (1987), Pape (2003), Van der Schoot et al. (2009) and Verschaffel et al. (1992) showed that children find it easier to convert the relation term 'more than' to a subtraction operation (as in the example above) than the relational term 'less than' to an addition operation. This effect has been explained by assuming that the semantic memory representation of 'less than' is more complex than that of 'more than', an effect which is known as the lexical marking principle (Clark, 1969). The reason behind this effect is that the marked relational term ('less than') and unmarked relational term ('more than') differ in their frequency of occurrence (**French, 1979; Goodwin & Johnson-Laird, 2005; Schriefers, 1990). Whereas the marked term is used only in its contrastive, 'negative' sense ('Peter has less marbles than David'), the unmarked term is used in two senses: the contrastive, 'positive' sense ('Peter has more marbles than David') but also a neutral, nominal sense ('Does she have more than one child?'). For word problem solving, the implication is that the memory representation of 'less than' is more 'fixed' than the memory representation of 'more than' (Van der Schoot et al., 2009).