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Review

Why is learning fraction and decimal arithmetic so difficult?

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ABSTRACT

Fraction and decimal arithmetic are crucial for later mathematics achievement and for ability to succeed in many professions. Unfortunately, these capabilities pose large difficulties for many children and adults, and students' proficiency in them has shown little sign of improvement over the past three decades. To summarize what is known about fraction and decimal arithmetic and to stimulate greater amounts of research in the area, we devoted this review to analyzing why learning fraction and decimal arithmetic is so difficult. We identify and discuss two types of difficulties: (1) inherent difficulties of fraction and decimal arithmetic and (2) culturally contingent difficulties that could be reduced by improved instruction and prior knowledge of learners. We conclude the review by discussing commonalities among three interventions that have helped children overcome the challenges of mastering fraction and decimal arithmetic.

Introduction

In 1978, as part of the National Assessment of Educational Progress (NAEP), more than 20,000 U.S. 8th graders (13- and 14-year-olds) were asked to choose the closest whole number to the sum of 12/13 + 7/8. The response options were 1, 2, 19, 21 and "I don't know". Only 24% of the students chose the correct answer, "2" (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). The most common answer was "19".

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This lack of understanding proved not to be limited to fraction arithmetic. The 1983 NAEP asked another large, representative sample of U.S. 8th graders to choose the closest whole number to the decimal arithmetic problem, 3.04 * 5.3. The response options were 1.6, 16, 160, 1600, and "I don't know". Only 21% of 8th graders chose the correct answer, 16 (Carpenter, Lindquist, Matthews, & Silver, 1983). The most common answer was "1600".

In the ensuing years, many efforts have been made to improve mathematics education. Governmental commissions on improving mathematics instruction (e.g., National Mathematics Advisory Panel, 2008), national organizations of mathematics teachers (e.g., National Council of Teachers of Mathematics, 2007), practice guides sponsored by the U.S. Department of Education to convey research findings to teachers (e.g., Siegler et al., 2010), widely adopted textbooks (e.g., Everyday Mathematics), and innumerable research articles (e.g., Hiebert & Wearne, 1986) have advocated greater emphasis on conceptual understanding, especially conceptual understanding of fractions. (Here and throughout the review, we use the term *fractions* to refer to rational numbers expressed in a bipartite format (e.g., 3/4). We use the term *decimals* to refer to rational numbers expressed in base-10 notation (e.g., 0.12)).

To examine the effects of these calls for change, we recently presented the above-cited fraction arithmetic question to 48 8th graders taking an algebra course. The students attended a suburban middle school in a fairly affluent area. Understanding of fraction addition seems to have changed little if at all in the 36 years between the two assessments. In 2014, 27% of the 8th graders identified "2" as the best estimate of 12/13 + 7/8. Thus, after more than three decades, numerous rounds of education reforms, hundreds if not thousands of research studies on mathematics teaching and learning, and billions of dollars spent to effect educational change, little improvement was evident in students' understanding of fraction arithmetic.

Such lack of progress is more disappointing than surprising. Many tests and research studies in the intervening years have documented students' weak understanding of fractions (e.g., Perle, Moran, & Lutkus, 2005; Stigler, Givvin, & Thompson, 2010). The difficulty is not restricted to the U.S. or to fractions. Understanding of multiplication and division of decimals also is weak in countries that are top performers on international comparisons of mathematical achievement, for example China (e.g., Liu, Ding, Zong, & Zhang, 2014; OECD, 2014).

Given the importance of knowledge of rational numbers for subsequent academic and occupational success, this weak understanding of fraction and decimal arithmetic is a serious problem. Early proficiency with fractions uniquely predicts success in more advanced mathematics. Analyses of large datasets from the U.S. and the U.K. showed that knowledge of fractions (assessed primarily through performance on fraction arithmetic problems) in the 5th grade is a unique predictor of general mathematic achievement in the 10th grade. This was true after controlling for knowledge of whole number arithmetic, verbal and nonverbal IQ, working memory, family education, race, ethnicity, and family income (Siegler et al., 2012). Other types of data have led to the same conclusion. For example, a nationally representative sample of 1000 U.S. algebra teachers ranked poor knowledge of "rational numbers and operations involving fractions and decimals" as one of the two greatest obstacles preventing their students from learning algebra (Hoffer, Venkataraman, Hedberg, & Shagle, 2007).

The importance of fraction and decimal computation for academic success is not limited to mathematics courses. Rational number arithmetic is also ubiquitous in biology, physics, chemistry, engineering, economics, sociology, psychology, and many other areas. Knowledge in these areas, in turn, is central to many common jobs in which more advanced mathematics knowledge is not a prerequisite, such as registered nurse and pharmacist (e.g., for dosage calculation). Moreover, fraction and decimal arithmetic are common in daily life, for example in recipes (e.g., if 3/4 of a cup of flour is needed to make a dessert for 4 people, how much flour is needed for 6 people), and measurement (e.g., can a piece of wood 36 inches long be cut into 4 pieces each 8.75 inches long). Fraction and decimal arithmetic are also crucial to understanding basic statistical and probability information reported in media and to understanding home finance information, such as compound interest and the asymmetry of percent changes in stock prices (e.g., the price of a stock that decreases 2% one month and increases 2% the next is always lower than at the outset).

Fraction and decimal arithmetic are also vital for theories of cognitive development in general and numerical development in particular. As with so many other topics, Piaget and his collaborators were probably the first to recognize the importance of understanding of rational number topics,