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# Defining and measuring conceptual knowledge in mathematics

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ABSTRACT

A long tradition of research on mathematical thinking has focused on procedural knowledge, or knowledge of how to solve problems and enact procedures. In recent years, however, there has been a shift toward focusing, not only on solving problems, but also on conceptual knowledge. In the current work, we reviewed (1) how conceptual knowledge is defined in the mathematical thinking literature, and (2) how conceptual knowledge is defined, operationalized, and measured in three mathematical domains: equivalence, cardinality, and inversion. We uncovered three general issues. First, few investigators provide explicit definitions of conceptual knowledge. Second, the definitions that are provided are often vague or poorly operationalized. Finally, the tasks used to measure conceptual knowledge do not always align with theoretical claims about mathematical understanding. Together, these three issues make it challenging to understand the development of conceptual knowledge, its relationship to procedural knowledge, and how it can best be taught to students. In light of these issues, we propose a general framework that divides conceptual knowledge into two facets: knowledge of general principles and knowledge of the principles underlying procedures.



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Research on mathematical thinking has typically divided mathematics knowledge into two types: procedural knowledge and conceptual knowledge (e.g., Hiebert, 1986).[1] In many mathematical domains, research has focused on procedural knowledge, typically defined as knowledge of sequences of steps or actions that can be used to solve problems (e.g., Rittle-Johnson & Siegler, 1998). In line with this theoretical definition, the way in which procedural knowledge is measured has become relatively standardized: participants solve a set of problems, and a score is calculated based on how many correct answers they obtain or based on the specific procedures they use to arrive at those answers. In recent years, however, the number of studies focused on procedural knowledge has been eclipsed by a growing literature on conceptual knowledge (see Star, 2005). There has been a shift toward studying, not only how people solve problems, but also their understanding of mathematical concepts, more broadly.

This shift in research, from a focus on procedures to a focus on conceptual knowledge, mirrors a similar trend in the mathematics education community. Mathematics curricula in the US have traditionally emphasized teaching children problem-solving procedures, with less emphasis on teaching the conceptual basis of the skills being learned (Stigler & Hiebert, 1999). However, recent reform efforts – as reflected, for example, in the standards from the National Council of Teachers of Mathematics and in the Common Core State Standards – have placed comparable emphasis on students having integrated conceptual and procedural knowledge (e.g., National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The general consensus, in research on mathematical thinking and in mathematics education, is that having conceptual knowledge confers benefits above and beyond having procedural skill.

The literature suggests a number of specific ways in which conceptual knowledge might prove useful. Some of the reported benefits connect directly to procedural skills. For example, conceptual knowledge has been shown to help people evaluate which procedure is appropriate in a given situation (e.g., Brownell, 1945; Byrnes & Wasik, 1991; Carr, Alexander, & Folds-Bennett, 1994; Garofalo & Lester, 1985; Greeno, 1978; Schneider & Stern, 2012). Conceptual knowledge also allows for more flexible problem solving, in that people who understand the conceptual underpinnings of a procedure are more likely to successfully generalize it to novel problems (e.g., Baroody & Dowker, 2003; Baroody, Feil, & Johnson, 2007; Blote, Klein, & Beishuizen, 2000; National Council of Teachers of Mathematics, 2000; Rittle-Johnson, Siegler, & Alibali, 2001). Once a problem has been solved, conceptual knowledge can also be used to check whether the solution is reasonable (e.g., Brownell, 1945; Carr et al., 1994; Garofalo & Lester, 1985).

It has also been suggested that conceptual knowledge provides more general benefits. The Common Core State Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), for example, explicitly mention that teaching conceptual knowledge in addition to procedures is a way to instill deeper and longer-lasting mathematical understanding. Thus, there is a widely held belief that conceptual knowledge plays an important role in mathematics learning.

Despite a clear movement in both research and educational practice toward emphasizing conceptual knowledge in addition to procedural knowledge, there are several obstacles standing in the way of a comprehensive understanding of conceptual knowledge. One major hurdle for researchers is that there does not appear to be a clear consensus in the literature as to what exactly conceptual knowledge is and how best to measure it. The term “conceptual knowledge” has come to denote a wide array of constructs, making it difficult to understand the major findings in the field, the ways in which conceptual knowledge relates to procedural knowledge, and the most effective ways to utilize current research to guide instructional practices (e.g., Baroody et al., 2007; Star, 2005). In particular, the diverse ways in which conceptual knowledge has been defined theoretically and the diverse ways in which it has been measured have created a wide-ranging literature in which a consistent “bigger picture” is hard to find.

[1] Some conceptualizations of mathematical knowledge include additional knowledge types (e.g., Rittle-Johnson & Koedinger, 2005) or divide knowledge into slightly different categories (e.g., Reason, 2003). The conceptual/procedural distinction, however, remains the dominant framework in the literature.