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# Exploring conceptions of infinity via super-tasks: A case of Thomson's Lamp and Green Alien

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ABSTRACT

In mathematics education research paradoxes of infinity have been used in the investigation of conceptions of infinity of different populations, including elementary school students and pre-service high school teachers. In this study the Thomson's Lamp paradox and a variation of it, the Green Alien, are used to investigate the naïve and emerging conceptions of infinity in a group of liberal arts university students, and the effect of context on such conceptions. We describe the difficulties that students face and the strategies they employ to make sense of the counterintuitive situations. This study contributes to research on the use of paradoxes in mathematics education and to research on understanding infinity, with a focus on infinitely small quantities.

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## 1. Introduction

The idea of infinity is likely one of the most challenging and intriguing in mathematics. However, it surfaces in a variety of different contexts, starting with enumerating natural numbers. Therefore, it is not surprising that students' conceptions of infinity have attracted interest of researchers in mathematics education. A variety of studies were conducted in an attempt to unravel students' struggle with seemingly counterintuitive results related to infinity. In particular, researchers have attended to comparing infinite sets (e.g. Tsamir & Tirosh, 1999), determining limits (e.g. Mamona-Downs, 2001), constructing infinite iterative processes (e.g. Brown, McDonald, & Weller, 2010), and considering infinity-related paradoxes (e.g. Dubinsky, Weller, McDonald, & Brown, 2005a,2005b; Mamolo & Zazkis, 2008). We contribute to this research by describing students' attempts in engaging with one particular paradox known as Thomson's Lamp. However, before introducing the paradox and the detail of our study, we provide a brief historical introduction of infinity and then turn to the use of paradoxes in mathematics education research, with a particular focus on paradoxes on infinity.

### *1.1. On counterintuitive notion of infinity*

Human beings have struggled with, reflected on and wondered about infinity since the beginning of recorded history. As Hilbert (1926) pointed out "From time immemorial, the infinite has stirred men's (sic) emotions more than any other question. Hardly any other idea has stimulated the mind so fruitfully. Yet, no other concept needs clarification more than it does" (p. 5). History shows that the ancient cultures had various ideas of infinity. In the 5th century B.C., Zeno of Elea devised paradoxes, known as Zeno's paradoxes, using infinite subdivision of space and time. Zeno's paradoxes continue to create discussion, debate and controversy even to this day. They highlight the counterintuitive nature of infinity. Aristotle

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(384-322 B.C.) introduced the dichotomy of potential infinity and actual infinity as a means of dealing with paradoxes of the infinite that he believed could be resolved by refuting the existence of actual infinity. One can think of potential infinity as a process, which at every instant of time within a certain time interval is finite. Actual infinity describes a completed entity that encompasses what was potential.

In the late nineteenth century George Cantor (1845–1918) developed a mathematical theory that explains certain aspects of infinity. Cantor's mathematical theory of cardinality of infinite sets is based on the very simple idea of one to one correspondence. Two sets *A* and *B* have the same cardinality or, more informally, the same size or the same number of elements if there is a one to one correspondence between them. But this idea leads to very counterintuitive results. Two infinite sets can have the same cardinality, or more informally the same number of elements, even though one is a proper subset of the other. In fact, an infinite set can be defined as a set that has a one to one correspondence with a proper subset of itself. While the notion of infinity introduces a variety of surprising and counterintuitive results, the surprise is explicitly highlighted when considering infinity-related paradoxes—to which we turn in what follows.

### *1.2. Paradoxes of infinity*

In mathematics education research paradoxes have been used as a lens on student learning. Movshovitz-Hadar and Hadass (1990 & 1991) investigated the role mathematical paradoxes can play in the pre-service education of high school mathematics teachers. They concluded that "a paradox puts the learner in an intellectually unbearable situation. The impulse to resolve the paradox is a powerful motivator for change of knowledge frameworks. For instance, a student who possesses a procedural understanding may experience a transition to the stage of relational understanding" (Moshovitz-Hadar & Hadass, 1991, p. 88). Commenting on their use of paradoxes in teaching they say that "it stems from a philosophy of teaching mathematics through errors, conflicts, debates, and discussions, that leads to gradual purification of concepts" (Moshovitz-Hadar & Hadass, 1991, p. 89). Sriraman (2008) used Russell's paradox in a 3-year study with 120 pre-service elementary teachers and studied their emotions, voices and struggles as they tried to unravel the paradox.

Recognizing the pedagogical value of paradoxes, and their usefulness as a research tool, several researchers focused on paradoxes of infinity. For example, Mamolo and Zazkis (2008) used the Hilbert's Grand Hotel paradox and the Ping-Pong Ball Conundrum to explore the naive and emerging conceptions of infinity of two groups of university students with different mathematical backgrounds. Núñez (1994) used Zeno's paradox, the Dichotomy, in a progressive manner to investgate how the idea of inifnity in the small emerges in the minds of students aged 8, 10, 12, and 14. Núñez (1994) concluded that conceptions of infinity in the large and infinity in the small are very different, especially for young learners. For example, though 8 years olds can conceive of the notion of endless in their consensual world they cannot see "infinity in the small". We extend these studies, focusing on paradoxes and infinity in the small, by investigating university students' reactions to particular super-tasks: Thomson's Lamp paradox and one of its variations.

### *1.3. **Thomso**n's Lamp*

Paradoxes such as Achilles and the Tortoise and the Ping-Pong Ball Conundrum are super-tasks. "A super-task may be defined as an infinite sequence of actions or operations carried out in a finite interval of time" (Laraudogoitia, 2011). Thomson (1954) devised the following paradox, known as the Thomson's Lamp, to show that super-tasks are impossible:

Think of a reading lamp with an on/off switch button. Suppose the button can be pressed in an instant of time. Suppose at the start the lamp is off. After 1 min the button is pressed and the lamp is on. After another 1/2 min the button is pressed and the lamp is off. After another 1/4 min the button is pressed and the lamp is on, and so on. That is, the switch is pressed and the on/off position of the lamp is flipped when exactly one-half of the previous time interval elapses. At the end of the two minutes, is the lamp on or off?

Thomson (1954) argued that the lamp cannot be on as it is never turned on without turning it off some time later. And the lamp cannot be off as it is never turned off without turning it on some time later. Therefore a contradiction arises, and since this super-task is kinematically impossible, it is logically impossible to determine the state of the Thomson's lamp at the end of the time interval. The "impossibility argument", has resulted in considerable discussion among mathematicians and philosophers alike (e.g., Laraudogoitia, 2011). For example, Benacerraf (1962) argued that the above reasoning is valid only for any instant between 0 and 2 min, but not at exactly 2 min.

Núñez (1994) noted that in the Dichotomy (Zeno's paradox) there are two attributes that one has to iterate simultaneously, the number of steps one has to make and the distance that each of these steps cover. Similarly, there are two attributes that one has to iterate simultaneously in the Thomson's Lamp Paradox, the number of half time intervals and the duration of each half time interval. As the number of half time intervals increases, the duration of half time intervals decreases and it converges to zero. Coordination of these two attributes is a potential source of difficulty for a learner. We were interested in exploring how students address this difficulty and what explanation they provide in discussing the presented super-tasks.

### *1.4. Theoretical considerations*

As stated, conceptions of infinity in general, and those related to paradoxes in particular, are challenging for learners. As a possible way to deal with a challenging idea, learners may attempt to access it at a lower level of abstraction than is required

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