Journal of Mathematical Behavior 41 (2016) 191–209

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Learning to represent, representing to learn

Sarah Kate Selling*

University of Michigan, United States

ARTICLE INFO

Article history:
Received 16 July 2014
Received in revised form 31 August 2015
Accepted 6 October 2015
Available online 2 November 2015

Keywords:
Representation
Mathematical practices
Collaboration
Agency
Algebra

ABSTRACT

This study explores how students learn to create, discuss, and reason with representations to solve problems. A summer school algebra class for seventh and eighth graders provided opportunities for students to create and use representations as problem-solving tools. This case study follows the learning trajectories of three boys. Two of the three boys had been low-achievers in their previous math classes, and one was a high achiever. Analysis of all three boys' written work reveals how their representations became more sophisticated over time. Their small group interactions while problem-solving also show changes in how they communicated and reasoned with representations. For these boys, representation functioned as a learning practice. Through constructing and reasoning with representations, the boys were able to engage in generalizing and justifying claims, discuss quadratic growth, and collaborate and persist in problem-solving. Negotiating different student-constructed representations of a problem also gave them opportunities to act with agency, as they made choices and judgments about the validity of the different perspectives. These findings have implications for the importance of giving all students access to mathematics through representations, with representational thinking serving as a central disciplinary practice and as a learning practice that supports further mathematics learning.

1. Introduction

Mathematical objects and relationships are abstract. We understand and operate on them only through representations of these ideas. For example, a relationship between two quantities is an abstract idea, but it can be represented using words, algebraic symbols, a table, or a graph. We use representational forms to communicate ideas and as tools for reasoning. Therefore, mathematical proficiency hinges on learning how to construct, communicate, and reason with representations (RAND, 2003). In mathematics classrooms, representations are often treated solely as the product of mathematics questions (Greeno & Hall, 1997). When students solve countless problems asking to "graph a function" or "make an *x*-*y* table," this sends the message that these representations are merely solutions to problems. This contrasts with authentic mathematical activity in which representations are constructed when useful as tools for thinking and communicating about mathematics (Bass, 2011; Pickering, 1995; Singh, 1998). This study seeks to contribute to our understanding of how students learn to create representations in mathematically authentic ways, as well as to understand the ways in which 'learning to represent' might play a critical role in mathematics learning.

* Correspondence to: School of Education, University of Michigan, 610 E University Ave, Ann Arbor, MI 48109, United States.
E-mail address: sselling@umich.edu

http://dx.doi.org/10.1016/j.jmathb.2015.10.003

2. Representations and representational practice

Representations have long been studied in relation to their importance in mathematics learning (Goldin, 1998; Kaput, 1998; Polya, 1981), as well as in the work of research mathematicians (Bass, 2011; Pickering, 1995; Singh, 1998). Dreyfus (1991) argues that learning progresses through four stages: using one representation, using multiple representations in parallel, making connections between parallel representations, and finally integrating representations and moving flexibly between them. Similarly, Duval (1999) argues that learning mathematics depends on the coordination of representational forms. Empirical studies have also provided evidence of the value of learning mathematics with multiple representations (e.g., Brenner et al., 1997; White & Pea, 2011).

Because representation is so important to mathematics, students need opportunities to learn how to engage in it. This is reflected in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics*, which includes a process standard for representation, asking students to: "create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena" (2000, pp. 67–71). The Common Core Standards also include representational activities that are integrated into the Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In both of these documents, representation is presented as more than what students should be able to produce (e.g. a graph or a table). Representation is an activity; it is a process or practice that students should do while learning and doing mathematics.

Several scholars have built theoretical arguments for this practice-oriented perspective on representation, including Latour and Woolgar's (1979) seminal study of scientists at work. More recently, Roth and McGinn (1998) provide a powerful argument for treating representation as a social practice by focusing on inscriptions: written or physical artifacts that represent something else (the referent). The authors argue that the relationship between inscriptions and their referents is not a matter of truth; instead, the relationship is a matter of social practice. This means that students need "to appropriate the use of inscriptions by participating in related social practices" (p. 54). They conclude that learning environments should be entered around the production and use of student inscriptions, rather than the transfer of canonical inscriptions (e.g., Cartesian graphs) from the teacher to students. Similarly, Greeno and Hall (1997) argue that learning to construct and interpret representations means learning to participate in the "complex practices of communication and reasoning in which the representations are used" (p. 362). They acknowledge that it is important for students to become familiar with standard forms of representation; however, they also call for students to become actively involved in constructing and discussing non-standard representations of mathematical ideas.

DiSessa (2000, 2004) has also examined representation as a social practice, through unpacking what he terms meta-representational competence (MRC), which involves higher-level skills, such as inventing new representations, explaining representations, or critiquing/comparing representations (p. 293). In a series of design studies, groups of students worked to invent representations of motion and landscapes. He found that students drew on their competence for MRC to both construct a rich variety of different representations and to critique the representations that others created. He argues that this competence does not refer to innate abilities; rather, it is developed through cultural practices in the children's lives. These studies show that students are capable of engaging in representation in a wide variety of inventive ways.

Empirical studies have also demonstrated the value and challenge of treating representation as a social practice in the math classroom. Moschkovich (2008) documented how the multiple interpretations of an inscription in a classroom provided valuable resources for discussion of important mathematical ideas. Cobb (2002) demonstrated how reasoning with tools and inscriptions was central to the development of a classroom mathematical practice. Forman and Ansell (2002) also examined the role of inscriptions in classroom mathematical discourse. They claim that the classroom discourse community came to resemble a scientific community when students had opportunities to construct arguments about inscriptions in classroom discussions. However, they cite the challenges involved in orchestrating the multiple voices of different students in these discussions.

A twelve-year longitudinal study conducted by a team of researchers from Rutgers has also yielded promising findings regarding students' development in representational practice. Speiser, Walter, and Maher (2003) present a case of high school students working with a wide range of representations. Their analysis demonstrates how the students were able to make sense of motion through reasoning with representations. Warner, Schorr, and Davis (2009) document the evolution of "representational flexibility" through rich cases of students inventing, tinkering with, and modifying representations. Maher, Powell, and Uptegrove (2010) trace the development of a particular mathematical strand, combinatorics, from 2nd grade through high school. Students constructed and shared representations as they worked to solve problems. Through the years, they modified and extended their representations in ways that helped them develop complex reasoning and justification practices. Furthermore, with the support of these representations and justification skills, students were able to identify the similar underlying mathematical structure in seemingly different problems.

Creating and negotiating different representations in the classroom can also position students to develop what Pickering (1995) describes as conceptual (or human) agency, which involves creating initial ideas or extending already established ones. See both Boaler and Sengupta-Irving (2016) and Fiori and Selling (2016) for a detailed analysis of the role of agency in mathematics classrooms. Greeno (2011) explored the relationship between agency and representation through analyzing video records of students engaged in an activity to develop representational practice. He differentiated between two aspects of conceptual agency: "generating variation (problematizing) and for contributing to selecting which alternatives