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Individual differences in children's approximations of area correlate with competence in basic geometry

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ABSTRACT

This study concerns the links between analog representations of spatial extent and visuospatial geometric competence in childhood. Research with college students suggests that individual differences in adults' ability to approximate cumulative surface area are reflected in their knowledge of school-relevant geometry. Other research suggests that the link between analog magnitude representations and mathematical reasoning may be present earlier in development, at least for representations of numerical magnitude and arithmetic concepts. Here we asked whether the understanding of basic geometric concepts and transformations, such as parallelism and mental rotation, makes similar psychological connections to spatial magnitude, particularly representations of area, prior to extensive education with geometry. We found that the precision with which 4- to 6-year-olds approximate area for irregular 2D shapes positively correlated with their performance on a basic geometry test, even when controlling for age and verbal intelligence. This finding points to a previously undocumented relationship between a system for representing analog magnitude and geometric competence. The potential interactions with systems of object recognition and navigation are discussed.

1. Introduction

Mathematics is a constellation of quantitative concepts and operations that span different content areas such as arithmetic, algebra, and geometry. As a product of human culture, the learning of formal math takes many years of education. Yet accumulating evidence suggests that basic aspects of mathematical understanding may be present early in development, prior to explicit instruction. Preschool children, and even infants, perform rudimentary arithmetic computations (Barth, Beckmann, & Spelke, 2008; Simon, Hespos, & Rochat, 1995; Wynn, 1992) and display sensitivity to geometric properties (Cohen & Younger, 1984; Izard & Spelke, 2009; Lourenco & Huttenlocher, 2008; Slater, Mattock, Brown, & Bremner, 1991). Researchers have speculated about the perceptual and cognitive capacities that underlie formal math competence, particularly in the domain of arithmetic. Such capacities include parallel individuation (Carey, 2004; Feigenson, Dehaene, & Spelke, 2004), set binding (Butterworth, 2010; Halberda, Sires, & Feigenson, 2006), an appreciation of the successor function (Leslie, Gelman, & Gallistel, 2008), and, the focus of the current study, representations of analog magnitude (Gallistel & Gelman, 1992; Halberda, Mazzocco, & Feigenson, 2008).

Number and other magnitudes such as spatial extent can be mentally represented in analog format (Buckley & Gillman, 1974; Gallistel & Gelman, 2000; Pinel, Piazza, Le Bihan, & Dehaene, 2004). Most models of analog magnitude depict representations of individual values as (Gaussian) distributions of activation along an internal continuum of values (Dehaene & Changeux, 1993; Gallistel & Gelman, 1992). Analog magnitudes are thus inherently imprecise, with imprecision increasing in proportion to the objective value, such that representations of larger values are noisier than smaller values. A behavioral signature of such imprecision is that discrimination of two magnitudes varies as a function of their ratio; when comparing, for instance, two numbers, accuracy decreases as the ratio approaches 1 (e.g., 8 vs. 4 compared with 10 vs. 9). Studies from comparative psychology suggest that human and nonhuman animals discriminate numerical and non-numerical magnitudes according to Weber's law, consistent with analog format (Ansari, 2008; Cantlon & Brannon, 2006; Nieder & Dehaene, 2009; Tudusciuc & Nieder, 2007). Although visual cortex (e.g., V1) is known to code for basic spatial properties (e.g., orientation and edge detection; Boynton, Demb, Glover, & Heeger, 1999; Schoups, Vogels, Qian, & Orban, 2001), representations of analog magnitude with their characteristic ratio limits have been found to emerge in parietal cortex, particularly the intraparietal sulcus (IPS) of humans and the non-human primate homolog for magnitudes such as spatial extent (e.g., lines of different length) and non-symbolic number (Jacob & Nieder, 2009; Pinel et al., 2004; Tudusciuc & Nieder, 2007). There is also evidence that infants discriminate visual stimuli on the basis of overall size (Baillargeon & Devos,

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1991) and, similar to numerical magnitude, these discriminations abide by Weber's law (Brannon, Lutz, & Cordes, 2006). As is the case with analog representations of number, children become better able to detect smaller differences in spatial magnitudes with age, demonstrating that analog representations of spatial extent also increase in precision over development (Odic, Libertus, Feigenson, & Halberda, 2013).

There has been growing interest in the malleability of analog magnitude representations as well as their potential links to uniquely human mathematics. Experiments comparing the precision of non-symbolic number approximation with knowledge of symbolic number concepts and arithmetic competence point to significant correlations in adults (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012) and school-age children (Halberda et al., 2008; Mazzocco, Feigenson, & Halberda, 2011). In these studies, participants who approximate the numerosity of a set of objects more accurately, and hence have more precise representations of numerical magnitude, score higher on tests of math competence. It should be noted, however, that not all studies have found this association (Sasanguie, Defever, Maertens, & Reynvoet, 2014; Iuculano, Tang, Hall, & Butterworth, 2008), creating a controversy over whether there is a direct link between analog magnitude representations and math competence, and, if so, what mechanisms underlie this link. Some researchers suggest that the association is mediated by an understanding of symbolic number concepts (Holloway & Ansari, 2009; Lyons & Beilock, 2011) or other cognitive processes such as inhibitory control (Fuhs & McNeil, 2013; Gilmore et al., 2013). A recent meta-analysis, however, suggests that the precision of numerical magnitude maintains an independent, albeit modest, contribution to mathematical competence (Fazio, Bailey, Thompson, & Siegler, 2014).

Motivated by questions about the nature and breadth of the connections between analog magnitude representations and mathematical competence, Lourenco et al. (2012) examined whether the precision of non-numerical magnitude was predictive of geometric understanding, as measured by a standardized test of school-relevant geometry. In separate discrimination tasks, adult participants (college students) judged which of two dot arrays was larger in either number or spatial extent (i.e., cumulative area). Participants' performance on each task, which was modulated by the ratio of the arrays being compared, was positively correlated with their performance on the geometry test, even after accounting for non-mathematical (verbal) intelligence, though only the precision of area representations accounted for unique variance in geometric competence. Thus, individual differences in spatial magnitude related to understanding of geometric concepts such as parallel lines and angular size.

Though crucial for establishing a link between analog magnitude representations and geometric competence, the work by Lourenco et al. (2012) leaves open at least two important questions. One question is whether a similar relationship exists earlier in development. Such a question is important for understanding the origins of the links between systems for representing analog magnitude and for supporting geometric knowledge. The question of developmental origins is also important because it allows for determining the extent to which experience, particularly school instruction, may play a role in shaping the links between analog magnitude and geometry. If geometric concepts are rooted in analog magnitude representations, then the obvious prediction is that there should be a correlation between these two systems in young children. Another question is whether all geometric concepts benefit from analog magnitude representations. Geometry is not a monolithic domain. There are numerous types of geometric concepts that may or may not benefit from greater precision of analog magnitude. It is thus critical to consider the links to analog magnitude for different geometric concepts.

One reason to predict that a link between analog magnitude and geometric competence should exist in children is that spatial magnitude may play a role in learning geometry by grounding abstract geometric concepts and in supporting geometry-relevant computations in magnitude cues. A parallel to this possibility is work suggesting that numerical symbols such as number words and Arabic digits acquire meaning early in development when they are mapped onto pre-existing representations of analog numerical magnitude (Gelman & Gallistel, 2004; Leslie et al., 2008). Concepts such as parallelism and right angles are not independent of their spatial extent properties. Knowing that lines are parallel versus non-parallel requires discrimination of the distances between lines. Sharper representations of spatial magnitude could bootstrap the acquisition of concepts such as parallelism because what is considered parallel depends on the analog properties of the figures themselves. The less precise one's magnitude representations, the less likely one may be able to characterize parallel versus non-parallel because this is based on distinguishing constant distances from those that are not. Similarly, characterizing forms on the basis of global shape may recruit analog magnitude representations because distinguishing across shape category such as square versus rectangle involves subtle discriminations in line length, and distinguishing within shape category such as equilateral versus obtuse triangle may involve subtle discriminations in angular size. Following from this, we would predict that there should be a correlation between analog magnitude precision and geometric knowledge starting early in development because of the potential role of analog magnitude in learning geometry. Sharper magnitude representations could give children a stronger grip on these geometric concepts early in development, when they are first attempting to master the underlying meaning of the concepts.

However, an alternative possibility is that no such link exists. Analog magnitude representations could be irrelevant to geometric concepts such as global shape because characterizing a form on the basis of its shape requires abstraction from properties such as overall size. Indeed, to understand the equivalence across shapes, one must ignore such differences in spatial extent. For instance, small and large triangles may be different in size, but they are equivalent in shape. Moreover, others have proposed that humans' geometric intuitions build on systems of object recognition and navigation (e.g., Spelke, Lee, & Izard, 2010; Spelke & Lee, 2012), which may leave little, if any, room for a role of analog magnitude representations in supporting geometric knowledge. In this case, there would be no relationship between the precision of children's analog magnitude representations and geometric knowledge. On this view, the emergence of a relationship between these systems in adults (Lourenco et al., 2012) might follow exposure to school instruction with mathematics. Formal instruction with geometry in particular could serve to establish the connection to a system of analog magnitude representation by explicitly highlighting the relevance of magnitude to geometric concepts and computations (Lourenco et al., 2012). For instance, geometry curricula that emphasize how individual blocks combine to create different shapes might highlight the importance of area representations (Clements, Wilson, & Sarama, 2004). Geometry instruction also incorporates formal computations such as calculating the area of a shape. These computations, which have been shown to relate to analog number representations (Lourenco et al., 2012), could highlight the relevance of analog magnitude pertaining to spatial extent for geometric knowledge.

In the current study we tested whether analog representations of spatial extent were correlated with children's understanding of visuospatial geometric concepts such as parallelism, angles, and global shape, as well as geometrically relevant transformations such as mental rotation, prior to extensive schooling with geometry specifically and mathematics more generally. To this end, we used a discrimination task in which 4- to 6-year-olds judged which of two irregular-shaped figures was larger in area by quickly approximately the overall size of each image. Similar to other discrimination tasks that have been used to measure analog magnitude precision (Leibovich & Henik, 2013; Lourenco et al., 2012; Odic, Libertus, Feigenson, & Halberda, 2013), ratio (for pairs of figures) was varied across trials (max/min: 2.00 to 1.11), creating trials that were more or less difficult to differentiate on the basis of area. We used irregular shapes in this task as in previous research to ensure that judgments were based on analog magnitude (i.e., area) rather than local spatial properties