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The improved grey model based on particle swarm optimization algorithm for time series prediction

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ABSTRACT

Grey theory is one of the most common methods for solving uncertain problems using limited data and poor information, due to its high performance in time series prediction. However, the inappropriate background value and initial value are the main factors affecting prediction accuracy of the Grey Model GM(1,1). An improved grey model based on particle swarm optimization algorithm named PGM(1,1) is proposed for time series prediction in this paper. The development coefficient of the grey model is calculated by PGM(1,1) based on particle swarm optimization, targeting at minimizing the average relative errors between the restored value and real value of the model to avoid the problem caused by background value optimization. In addition, the initial value of the Grey Model GM(1,1) is optimized and a sliding window is introduced to improve both precision and adaptability. Finally, three data sets, featuring increasing trend, decreasing trend, and wide fluctuations, are used in the experiments, showing that the proposed method achieves better prediction accuracy.

1. Introduction

Time series prediction is an important and widely interesting topic in the research of system modeling (Chunshien and Jhao-Wun, 2012). In real life, time series are often produced by a stochastic process that is often used in modeling time series data. Probability statistics, which is typically used to process such data, requires a large amount of data, making it difficult to yield accurate results when operating on limited data. Grey theory (Deng, 1982), which is used to solve uncertain problems with limited data and poor information, focuses on the construction of grey forecast model using a small amount of information. Instead of seeking the statistical law of the time series data, the grey theory relates the random process to time and utilizes accumulated generating operation to process raw data. The randomness inherent in data is reduced, the ruleless data is converted into exponential-like form, sequences with strong regularity are generated, and the future trend of the data can be predicted by the theory. Grey system is typically used to find the laws hidden in chaotic data.

The grey theory focuses on processing systems with limited sample size and partially known information (Xiao et al., 2013). It describes and predicts the variation of the data by analyzing known information and extracting valuable information. Being easy to implement and simply structured, it is widely used in natural science, social science, economic sphere, various industrial fields (Hui et al., 2015), engineering and science, etc. (Mao et al., 2012; Jiang et al., 2014). Grey Model GM(1,1), an important model used by Grey theory, requires a relatively small amount of data (typically four or more samples) to establish a prediction model and employs a simple calculation process to achieve high accuracy prediction. However, the prediction accuracy of model can be improved.

The improvements to the prediction accuracy of GM(1,1) mainly focus on two areas. The first focuses on the prediction precision by improving the selection of initial condition in the time response function. Instead of taking $X^{(0)}(1)$ as the initial value, the improved initial condition reselects the initial value to reduce error based on theoretical analysis (Luo et al., 2008); Wang et al. (2010) proposed a novel approach to improve prediction accuracy of GM(1,1) model through optimization of the initial condition. The proposed initial condition consists of the first and last items of a sequence generated by applying the first-order accumulative generation operator on the sequence of raw data.

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The second focuses on the prediction precision by improving the selection of the background value. Shan et al. (2013) proposed an improved model that accumulates generation sequence based on the GM(1,1) model. By employing a new formulation of background value, the new model achieves better fitting accuracy. These improvements for grey models increase the prediction accuracy in some practical applications. However, prediction accuracy of GM(1,1) model can be further improved.

In recent years, genetic algorithm and other intelligent algorithms have been used to improve the accuracy of GM(1,1) model. Yang (2012) proposed a method for solving the optimal values of the parameters and the optimal background correction term boundary value based on genetic algorithm. Xie et al. (2000) applied genetic algorithms to solve the value of λ and forecast the prime data according to λ. Since these improvements still use the least squares method to solve the time response parameters, most improved methods that are based on intelligent algorithms only optimize the background value.

In this paper, a new method that avoids constructing the background value and selects the initial value reasonably for optimizing the GM(1,1) model based on the particle swarm optimization algorithm is proposed. The development coefficient of the grey model based on particle swarm optimization is introduced in the proposed method, minimizing the average relative errors between the restored value and real value of the model. In addition, the initial value of the GM(1,1) model is optimized and sliding window is introduced to improve both accuracy and adaptability.

The rest of the paper is organized as following. Section 2 briefly introduces the traditional GM(1,1) model and particle swarm optimization algorithm. Section 3 describes the improved grey model based on particle swarm optimization. Section 4 illustrates the application of new optimized model using three numerical examples. Section 5 concludes with a summary of the contributions and future work.

2. Fundamental research

2.1. Particle swarm optimization algorithm

Particle swarm optimization (PSO), first proposed by Kennedy and Eberhart (1995), is a swarm intelligence optimization algorithm featuring high search speed and high efficiency. It is widely used in parameter optimization involved in various continuous and discrete problems.

In a D-dimensional search space, the population consisting of n particles is defined as $X = (X_1, X_2, \ldots, X_n)$; the position of the i-th particle in the population is defined as $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})$; the speed of the i-th particle is defined as $v_i = (v_{i1}, v_{i2}, \ldots, v_{id})$; the best position of the i-th particle has been searched is defined as $p_i = (p_{i1}, p_{i2}, \ldots, p_{id})$; the best position of the whole population has been searched is defined as $g_i = (g_{i1}, g_{i2}, \ldots, g_{id})$. In the iterative search process, each particle updates its speed and position based on individual and global extremums. The change of the speed and position of each particle can be expressed by the following formulas:

$$v_{id} = wv_{id} + c_1r_1(g_{id} - x_{id}) + c_2r_2(p_{id} - x_{id}). \tag{1}$$

$$x_{id} = x_{id} + v_{id}. \tag{2}$$

where c_1, c_2 are the acceleration factors that control the speed of the particles; r_1, r_2 are two random numbers independently generated in the range of [0,1]; w is the inertia weight factor. Each particle tracks its previous best position and the global best position by constantly updating its speed and location until the number of iterations exceeds the maximum or a standard error is reached. The basic steps of the PSO algorithm are demonstrated as follows (Pinkey et al., 2015):

(1) Initialize particle populations randomly and get the speed and position of all particles in the population.
(2) Evaluate all particles according to the defined fitness function based on the optimization goals.
(3) Update the individual extremum of each particle according to the fitness function.
(4) Update the global extremum of each particle according to the fitness function.
(5) Update the speed and position of the particle according to Eqs. (1) and (2).
(6) Repeat steps 2–5 until the number of iterations exceeds the maximum or a standard error is reached.

2.2. The original GM(1,1) model

The modeling process of traditional GM(1,1) grey prediction model is demonstrated as follows (He and Song, 2005):

(1) Assume that $X^{(0)}=\{x^{(0)}(k)\}$, $(k=1,2,\ldots n; n \geq 4)$ is a non-negative sequence of raw data, and its sequence of the first order accumulative generation operator on $X^{(0)}$ (1-AGO) is $X^{(1)}=\{x^{(1)}(k)\}$, where:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i). \tag{3}$$

(2) The sequence of generated mean value of consecutive neighbors can be derived by $Z^{(1)}=\{z^{(1)}(k)\}$, $(k=1, 2, 3,\ldots, n)$, where

$$Z^{(1)}(k) = \lambda x^{(1)}(k-1) + (1-\lambda)x^{(1)}(k),\ \lambda \in [0, 1]. \tag{4}$$

(3) Assume that $X^{(1)}$ has an approximately exponential variation, the whitened equation of GM(1,1) model can be expressed as:

$$\frac{dX^{(1)}}{dt} + aX^{(1)}(t) = b. \tag{5}$$

By discretizing Eq. (5), the grey differential equation, also known as GM(1,1) model, can be expressed as:

$$x^{(0)}(k) + az^{(1)}(k) = b. \tag{6}$$

(4) Calculate the values of parameters a and b (or A and B) by the least square estimation method:

$$A = \left(B^TB\right)^{-1}B^TY_N. \tag{7}$$

where

$$A = \begin{bmatrix} a \\ b \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ \ldots & \\ -z^{(1)}(n) & 1 \end{bmatrix}, = Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \ldots. \\ x^{(0)}(n) \end{bmatrix}$$

(5) Calculate the time response equation of GM(1,1) based on parameters a and b, shown as:

$$\widehat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak} + \frac{b}{a}. \tag{8}$$

(6) The restored values of raw data is given by equation:

$$\widehat{x}^{(0)}(k+1) = \widehat{x}^{(1)}(k+1) - \widehat{x}^{(1)}(k). \tag{9}$$

The prediction accuracy of GM(1,1) model depends on the