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Connectivity reliability in uncertain networks with stability analysis

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A B S T R A C T

This paper treats the fundamental problems of reliability and stability analysis in uncertain networks. Here, we consider a collapsed, post-disaster, traffic network that is composed of nodes (centers) and arcs (links), where the uncertain operationality or reliability of links is evaluated by domain experts. To ensure the arrival of relief materials and rescue vehicles to the disaster areas in time, uncertainty theory, which neither requires any probability distribution nor fuzzy membership function, is employed to originally propose the problem of choosing the most reliable path (MRP). We then introduce the new problems of α-most reliable path (α-MRP), which aims to minimize the pessimistic risk value of a path under a given confidence level α, and very most reliable path (VMRP), where the objective is to maximize the confidence level of a path under a given threshold of pessimistic risk. Then, exploiting these concepts, we give the uncertainty distribution of the MRP in an uncertain traffic network. The objective of both α-MRP and VMRP is to determine a path that comprises the least risky route for transportation from a designated *source node* to a designated *sink node*, but with different decision criteria. Furthermore, a methodology is proposed to tackle the stability analysis issue in the framework of uncertainty programming; specifically, we show how to compute the arcs' *tolerances*. Finally, we provide illustrative examples that show how our approaches work in realistic situation.

1. Introduction

Path finding and reliability problems have many applications in various real-life domains, such as transportation, traffic cybernetics, disaster management, message routing in communication systems, and production-distribution systems (Dietrich & Vohra, 1993; Hosseini, Sahin, & Unluyurt, 2014; Hosseini & Wadbro, 2015; Ji, Kim, & Chen, 2011; Ma, Tang, & Ke, 2008; Petrovic & Jovanovic, 1979; Xing & Zhou, 2011). In traffic cybernetics and communication networks, reliability is an essential measure of the quality of service for transportation, avoiding traffic congestion, and an important attribute in travelers' route (Ma et al., 2008). In disaster management, system reliability is the first issue to address by disaster managers for transmitting relief materials to the affected areas, and so a reliable path may ensure the arrival of these supplies (Wang, Zhu, & Yang, 2013).

Traffic networks, especially after extreme events such as earthquake, are subjected to a significant degree of uncertainty that may be attributed to factors such as travel time, link capacity, and system connectivity. This leads to adverse and unpredictable system performance reflected in a high degree of variability in system parameters. As a result, an uncertain network is a more realistic representation of an actual traffic network compared with the deterministic one. Moreover, in cases where no information of the links' functionality is available, it is impossible to estimate the probability distributions and thus to use random variables to describe the indeterminacy (Ding, 2015). In such cases, all available information is in the form of *belief degrees* given by some domain experts. However, dealing with belief degree by probability theory or fuzzy theory may result in counterintuitive outcome (Ding & Gao, 2014), since unlikely events are often overweighted and thus the variance of belief degree may be much larger than real frequency (Tversky & Kahneman, 1986).

To the best of our knowledge, there is no research addressing the most reliable path (MRP) problem and its stability analysis in the framework of *uncertainty theory*. Therefore, one of the motivations behind our study is to consider the *connectivity reliability problem* in an uncertain post-disaster traffic network in which the links' reliabilities are "unknown" or "not accurate enough." To ensure the arrival of relief materials and to dispatch rescue vehicles to disaster areas, we want to find the MRP between a source–sink pair. Here, the reliability of each link is an *uncertain variable*, which is neither fuzzy nor stochastic. Due to the uncertainty, we cannot find an MRP in the normal sense. Thus, we employ *uncertainty pro-*

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gramming to solve the MRP problem. In the context of traffic network, this kind of uncertainty is new and it neither requires any probability distribution nor fuzzy membership function. In this paper, we seek the MRP by minimizing the risk of losing path connectivity. We introduce the new problems of α*-most reliable path* (α-MRP) and *very most reliable path* (VMRP), and give the *uncertainty distribution* of the most reliable path.

Another contribution of this paper is to carry out further research in *stability analysis*, also called *sensitivity analysis*. Here, the aim is to determine how the MRPs are influenced by changes in various links' reliabilities, which yields the concept of link *tolerance*. We wish to determine, for each link in the network, what effect the corresponding link's reliability has on the MRP between two given nodes in the system. This paper assumes that the links' parameters are uncertain variables and presents stability analysis in traffic networks in the framework of uncertainty theory. We compute the links' tolerances with the aid of the *operational law* from uncertainty theory. We show that there exists an equivalence relation between the tolerances in an uncertain network and the tolerances of an α-MRP in an associated deterministic network. By using this relation, we obtain the distribution of the MRP and devise effective approaches to find the links' tolerances.

2. Literature review and motivation

Several attempts from operations research have been made over the last two decades to tackle network reliability problem both in communication networks and traffic/road networks. The existing reliability studies of road networks are mainly limited to three aspects, *connectivity reliability, travel time reliability*, and *capacity reliability*. Connectivity reliability is concerned with the probability that the network nodes remain connected. A special case is the terminal reliability, which concerns the existence of a path between a specific source–sink pair (Wang et al., 2013). Travel time reliability is concerned with the probability that a trip between a given source–sink pair can be made successfully within a specified interval of time. This measure is useful to evaluate network performance under normal daily flow variation (Chen, Skabardonis, & Varaiya, 2003; Clark & Watling, 2005; Xing & Zhou, 2011). Capacity reliability is a measure to evaluate the performance of a degradable road network (Chen, Yang, Lo, & Tang, 1999). It can also be considered as the probability that the network can accommodate a certain traffic demand at a specific service level (Wang et al., 2013).

During the last decade, there has been an increased interest to also include the stochastic nature of the network when addressing the path finding problem (Ji et al., 2011; Lee, Kim, & Jung, 2008; Ma et al., 2008; Srinivasan, Prakash, & Seshadri, 2014; Wang et al., 2013; Xing & Zhou, 2011). Most of these works address travel time reliability or capacity reliability (with different assumptions) using probability or fuzzy theories without a proper emphasis on the connectivity and stability. To the best of our knowledge, there has not been any works that have considered two important aspect of traffic networks, namely uncertain connectivity after disaster and the stability of paths under uncertainty. Motivated by this, we develop an uncertain most reliable path (UMRP) model that can accommodate connectivity requirements to ensure the arrival of relief materials and rescue vehicles to the post-disaster areas.

In recent years, optimization under uncertainty has occupied a central position in operations research. Humans and intelligent software agents are increasingly faced with the challenge of making decisions based on large volumes of streaming non-deterministic data from diverse sources (Arunkumar, Sensoy, Srivatsa, & Rajarajan, 2015). Uncertainty theory was founded by Liu in 2007 as a new branch of mathematics to deal with human uncertainty (Liu, 2010; 2015). During the last few years there has been a vast interest in developing strategies to solve problems in different fields with various uncertain phenomena, such as uncertain networks (Liu, 2010; 2015), uncertain shortest path (Gao, 2011), logistics system under supply and demand uncertainty (Moghaddam, 2015), multi-region supply chain under demand uncertainty (Langroodi & Amiri, 2016), uncertain graph and connectivity (Gao & Gao, 2013), uncertain multi objective traveling salesman problem (Wang et al., 2013), uncertain logic (Chen & Ralescu, 2011), uncertain inference control (Gao, 2012), uncertain multi-item supply chain network (Hosseini, 2015), uncertain multi-objective programming (>Wang, Guo, Zheng, & Yang, 2015), uncertain multi-product newsboy problem (Ding & Gao, 2014), uncertain maximum flow problem (Ding, 2015), and uncertain supplier selection problem (Memon, Lee, & Mari, 2015).

Furthermore, this paper also contributes to the growing body of knowledge regarding the sensitivity analysis. With the advances in networking and communication technologies, lots of interest has been given to sensitivity analysis of such systems and closely related problems, for example, shortest path and maximum capacity path problems (Ramaswamy, Orlin, & Chakravarti, 2005), minimum spanning tree or shortest path tree (Pettie, 2015; Shier & Witzgall, 1980; Tarjan, 1982), most reliable path (Chang & Amir, 2007), and reachability in uncertain linear systems (Kurzhanski & Varaiyab, 2005; 2006). Extensive references to a variety of stability analysis problems in combinatorial optimization can be found in Ahuja, Magnanti, and Orlin (1993), Gal (1995), and Jäger and Goldengorin (2014).

3. Preliminaries and uncertainty axioms

In many occasions, due to maintenance or technical difficulties, we have lack of observed data about an unknown state of nature. In this case, we have to invite some domain experts to evaluate their *belief degree* for events' occurrence. Recently, Liu (2010) proposed uncertainty theory to describe non-deterministic phenomena, especially expert data and subjective estimation. To indicate the belief degree that each event (links' failure) will occur, we need to deal with a *set function*. If a set function $\mathcal{M}$ satisfies the axioms of uncertainty theory A1–A5, listed below, then it is an *uncertain measure* and the links' failures are *uncertain variables*.

A1 $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ,
A2 $\mathcal{M}\{A\} + \mathcal{M}\{A^c\} = 1$ for any event $A \in \mathcal{L}$,
A3 $\mathcal{M}\{A_1\} \le \mathcal{M}\{A_2\}$ for any events A_1 and A_2 that satisfies $A_1 \subseteq A_2$,
A4 $\mathcal{M}\{\bigcup_{i=1}^{\infty} A_i\} \le \sum_{i=1}^{i=\infty} \mathcal{M}\{A_i\}$ for any sequence of events $\{A_i\}$,
A5 $\mathcal{M}\left\{\prod_{i=1}^{n} A_i\right\} = \min_{i \in \{1,\dots,n\}} \mathcal{M}_i\{A_i\}$ for uncertainty spaces $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i)$ and $A_i \in \mathcal{L}_i$.

The uncertain measure is interpreted as the personal belief degree (not frequency) of an uncertain event that may happen. It depends on the personal knowledge concerning the event. The uncertain measure will change if the state of knowledge changes. To rationally deal with belief degrees, let Γ be a nonempty set (sometimes, called *universal set*), and $\mathcal{L}$ be a σ-algebra over Γ. Then, the pair $(\Gamma, \mathcal{L})$ is called a *measurable space*, the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an *uncertainty space*, and each element $A \in \mathcal{L}$ is a *measurable set* that is referred to as an *event*. In order to present an axiomatic definition of *uncertain measure*, it is necessary to assign each event A a number $\mathcal{M}\{A\}$, which indicates the level that A will occur. Note that axiom A5 defines a *product uncertain measure* only for rectangles (Liu, 2015). Although probability measure satisfies the above first four axioms, it is not a special case of uncertainty theory because the product probability measure does not satisfy axiom A5. For more information, the reader is referred to the works by Liu (2010); 2015).

Having the uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ at hand, the *uncertain variable* ξ is defined as a measurable function from $(\Gamma, \mathcal{L}, \mathcal{M})$ to