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A multiobjective DEA approach to ranking alternatives

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ABSTRACT

The application of Data Envelopment Analysis (DEA) as a tool for efficiency evaluation has become widespread in public and private sector organizations. Since decision makers are often interested in a complete ranking of the evaluated units according to their performance, procedures that effectively discriminate the units are of key importance for designing intelligent decision support systems to measure and evaluate different alternatives for a better allocation of resources. This paper proposes a new method for ranking alternatives that uses common-weight DEA under a multiobjective optimization approach. The concept of distance to an ideal is thereby used as a means of selecting a set of weights that puts all the decision units in a favorable position in a simultaneous sense. Some numerical examples and a thorough computational experiment show that the approach followed here provides sound results for ranking alternatives and outperforms other known methods in discriminating the alternatives, therefore encouraging its use as a valuable decision tool for managers and policy makers.

1. Introduction

Data Envelopment Analysis (DEA), first introduced in Charnes, Cooper, and Rhodes (1978), is a mathematical programming technique useful for assessing the relative efficiency of a homogeneous set of decision making units (DMUs) in a production system with multiple inputs and multiple outputs. Subsequent developments have proved DEA as a valuable tool for performance evaluation in a wide number of fields, with interesting applications in health care, education, banking, manufacturing, etc. In the DEA methodology, for each DMU an efficiency score is computed as a ratio of a weighted sum of outputs to a weighted sum of inputs, with such set of weights found to guarantee the most favorable result for the DMU under evaluation. According to this score, every DMU is either found to perform efficiently or deemed inefficient, in which case DEA can find a corresponding set of efficient units to be used as a benchmark for improvement.

The problem tackled by DEA highly resembles the one studied within Multicriteria Decision Making (MCDM), in which a number of alternatives have to be evaluated and compared in terms of several conflicting criteria in order to achieve a ranking of the alternatives and/or select the best option. In fact, ranking a set of alternatives in real-world applications often turns into a rather overwhelming problem for many decision-makers that may require expertise and computational support, especially when the number of alternatives and criteria grow, and so the problem has been extensively studied. Particularly, the methodological connections existing between DEA and MCDM approaches (Belton & Vickers, 1993, Stewart, 1996) that become clear when we identify alternatives with DMUs and criteria with inputs and outputs, have led some authors to propose the use of DEA as a tool for MCDM (Doyle & Green, 1993, Mavrotas & Trifillis, 2006, Sarkis, 2000).

However, when using standard DEA techniques with a ranking purpose some difficulties may arise. First, DEA efficiency scores do not always allow a complete ordering of the alternatives since many of the DMUs are usually classified as efficient. The lack of discrimination in DEA applications is well documented, particularly when the number of inputs and outputs is too high relative to the number of DMUs being evaluated, and a number of techniques have been proposed to alleviate this drawback (Adler, Friedman, & Sinuany-Stern, 2002, Angulo-Meza & Lins, 2002, Hosseinzadeh Lofti et al., 2013). Moreover, it is argued that using different sets of weights is inappropriate in a ranking context because such flexibility deters the comparison among DMUs on a common base (Kao & Hung, 2005). Also, it has been noticed that some units classified as inefficient could in turn be better overall performers than certain efficient ones, possibly involved in an unreasonable weight scheme induced by the maximization of their own efficiency (Dyson & Thanassoulis, 1988).

The above reasoning suggests that, when DEA is being used with a ranking purpose, a common set of weights (CSW) is highly recommended in order to fairly expose all the units to the same

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light (Despotis, 2002) and thus provide a common base for the ranking. Several approaches have been proposed in the DEA literature to obtain a common set of weights (see Kao & Hung, 2005, Liu & Peng, 2008, Wang, Luo, & Lan, 2011b and references therein), including methods based on multivariate statistical analysis, regression analysis or cross-efficiency analysis, among others. However, the present research is mainly interested in those approaches based on a multiobjective view since they represent a natural generalization of the traditional DEA approach: the search of a DMU-dependent set of weights that is the most favorable to each individual DMU is generalized to the search of a common set of weights that is the most favorable to *all* the DMUs in a *simultaneous* way. This idea can be accomplished through the simultaneous maximization of the efficiency scores of all the DMUs, leading to a multiobjective programming problem that can be solved using a Compromise Programming approach (Despotis, 2002, Kao & Hung, 2005, Zohrehbandian, Makui, & Alinezhad, 2010). Some other authors approach the computation of the common weights by considering the simultaneous minimization of the differences between the weighted sum of inputs and the weighted sum of outputs for each DMU (Chen, Larbani, & Chang, 2009, Chiang, Hwang, & Liu, 2011). In this way they obtain a linear programming problem that is equivalent to the simultaneous maximization of efficiency ratios. Earlier methods built on a similar rationale are based on maximizing the average of efficiency ratios of all the units (Roll & Golany, 1993) or maximizing the minimum efficiency ratio (Troutt, 1997).

The objective of this research is to further investigate the application of DEA as a ranking tool within MCDM and therefore in the following sections a new DEA-based procedure for ranking alternatives is proposed which combines the CSW concept with the multiobjective paradigm that characterizes the above-mentioned approaches. More specifically, the concept of distance to an ideal is used to obtain a common set of weights that is favorable to all the alternatives simultaneously. The distance to an ideal DMU, defined as a hypothetical unit that consumes the least inputs to produce the most outputs, has been explored previously in a context of cross-efficiency evaluation, with each DMU choosing its own set of weights to minimize its distance from this ideal DMU (Wang, Chin, & Luo, 2011a) as well as in a ranking context (Jahanshahloo, Hosseinzadeh Lofti, Khanmohammadi, Kazemimanesh, & Rezaie, 2010, Sun, Wu, & Guo, 2013). However, the approach followed here differs from these former works since it does not rely on the definition of such an ideal unit, but on the computation of the ideal point in a 2-dimension space where the DMUs are mapped when the aggregate input and output measures are considered.

The rest of the paper is organized as follows. After this introduction, the main topics concerning MCDM and DEA approaches are briefly reviewed, with special attention to the most relevant CSW-DEA methods. Section 3 introduces a new DEA-based procedure for ranking a set of alternatives that combines the common-weight concept and the multiobjective methodology. The last sections illustrate the usefulness of the proposed approach and validate its application as a ranking tool within a multicriteria decision framework: in Section 4 some numerical examples are examined and Section 5 summarizes the results of the computational study performed. Finally, some concluding remarks are provided.

2. Methodological background

The term Multiple Criteria Decision Making is used to describe a collection of formal approaches that seeks to explicitly account for multiple conflicting criteria in the evaluation and comparison of a number of alternatives. Both MCDA and DEA have been receiving considerable attention in the specialized literature for the last 30 years, developing independently to each another during the first decades. However, after several authors established a number of analogies between MCDM and DEA methodologies (Belton & Vickers, 1993, Doyle & Green, 1993, Golany, 1988, Stewart, 1996), the two fields have been evolving in a more cooperative mode. In this work both approaches come together to solve a ranking problem.

First, some of the main concepts and techniques concerning MCDM and DEA are reviewed in this section.

2.1. MCDM preliminaries

The awareness of the multidimensional nature of socioeconomic phenomena and the need to consider more than a single criterion when judging them have encouraged the interest in the Multiple Criteria Decision Making paradigm. The multicriteria decision problem is mathematically defined as

$$\begin{aligned} &vopt \quad (z_1(x), z_2(x), \dots z_k(x)) \\ &s.\,t. \\ &\qquad\qquad x \in X \end{aligned} \tag{1}$$

where X represents the set of possible alternatives or feasible region, $x \in X$ is a n-dimensional vector containing decision variables, z_j are the objective functions representing the criteria that the decision-maker wants to attain and *vopt* stands for the simultaneous optimization of the k objective functions. In this context an optimal solution, which attains the optimum value of all the objectives, is generally impossible to find due to the conflictive nature of the criteria. Hence solving a multicriteria problem implies to find an *efficient* solution, which cannot be altered to improve one criterion without deteriorating at least another one.

Many different techniques are available for handling an arbitrary multicriteria decision problem (Steuer, 1986), a number of which essentially combine the multiple objectives into one single objective. Particularly popular is the weighting method, which attempts the optimization of a weighted sum of the k objectives, and the Compromise Programming method, where the solution with minimum L_p – distance to the ideal point, the one that comprises the optimal outcomes of all the objectives, is selected. This approach perfectly captures what is known as Zeleny's axiom of choice, stating that "alternatives that are closer to the ideal are preferred to those that are farther away" (Zeleny, 1982).

Then, if $z^* \in \mathrm{R}^k$ represents the coordinates of the ideal point, by considering the family of L_p metrics, the objective of choosing a solution as close as possible to the ideal is accomplished by

$$\begin{aligned} &min \quad \left(\sum_{i=1}^{k} \lambda_i \left(z_i^* - z_i(x) \right)^p \right)^{1/p} \\ &s.t. \\ &\qquad\qquad x \in X \end{aligned} \tag{2}$$

where λ_i is a weight associated with the ith objective function and the parameter $1 \le p \le \infty$ is related to the relative contributions of individual deviations. Particularly, when $p = 1$ the distance measures the sum of individual deviations over the k objectives, which is the longest distance between the two points in a geometric sense, or "Manhattan distance", when $p = 2$ the Euclidean distance to the ideal point is considered and ultimately, for $p = \infty$, the largest of the deviations completely dominates the distance measure. In this way, $p = 1$ represents total compensability among objectives and $p = \infty$ represents no compensability among objectives. Since other values of p are not easily interpreted, those are the most commonly used.

2.2. DEA preliminaries

Let us consider n production units or DMUs, each of them being evaluated in terms of r inputs and s outputs. Let x_{ij} and y_{ik} be nonnegative values denoting respectively the amount consumed