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A new belief rule base knowledge representation scheme and inference methodology using the evidential reasoning rule for evidence combination

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ABSTRACT

In this paper, we extend the original belief rule-base inference methodology using the evidential reasoning approach by i) introducing generalised belief rules as knowledge representation scheme, and ii) using the evidential reasoning rule for evidence combination in the rule-base inference methodology instead of the evidential reasoning approach. The result is a new rule-base inference methodology which is able to handle a combination of various types of uncertainty.

Generalised belief rules are an extension of traditional rules where each consequent of a generalised belief rule is a belief distribution defined on the power set of propositions, or possible outcomes, that are assumed to be collectively exhaustive and mutually exclusive. This novel extension allows any combination of certain, uncertain, interval, partial or incomplete judgements to be represented as rule-based knowledge. It is shown that traditional IF-THEN rules, probabilistic IF-THEN rules, and interval rules are all special cases of the new generalised belief rules.

The rule-base inference methodology has been updated to enable inference within generalised belief rule bases. The evidential reasoning rule for evidence combination is used for the aggregation of belief distributions of rule consequents.

1. Introduction

Rule-based knowledge representation is one of the most common schemes for representing various types of knowledge (Davis, 1986; Hayes-Roth, 1985). Moreover, it has been argued that other knowledge representation schemes can be transformed to rule-based (Nilsson, 1982; Sun, 1995). Rule-based systems are usually constructed from human knowledge in the form of IF-THEN rules and have been widely applied in fields of artificial intelligence and decision support systems (Azibi & Vanderpooten, 2002; Ligeza, 2006; Negnevitsky, 2005).

Traditional rule-based systems use simple IF-THEN rules to represent human expert knowledge. Traditional IF-THEN rules take the form of 'IF *P* THEN *Q*'. The consequent (*Q*) in the previous example is believed to be 100% true given that the antecedent (*P*) has happened. Previous research has shown that such strict knowledge representation scheme leaves no room for uncertain or incomplete judgements (Chen et al., 2012; Kong et al., 2012; Li, Wang, Yang, Guo, & Qi, 2011; Xu et al., 2007; Zhou, Hu, Xu, Yang, & Zhou, 2011). Therefore, it cannot be applied to domains where uncertain or incomplete knowledge is involved.

Limitations of traditional IF-THEN rules have attracted numerous researchers. Hall, Blockley, and Davis (1998) used interval probability theory, introduced by Cui and Blockley (1990) as a measure of evidential support in knowledge-based systems, for uncertain rule inference. Interval numbers were used to represent the probability measure in order to capture fuzziness and incompleteness. Yang, Liu, Wang, Sii, and Wang (2006) introduced *belief rules* as an extension to traditional IF-THEN rules by replacing single-valued consequents with a distributed assessment called *belief structures*. The use of distributed assessments has enabled various types of information to be incorporated into a decision making process without pre-aggregation (Zhang, Yang, & Xu, 1989). Elouedi, Mellouli, and Smets (2000, 2001); Nguyen and Goodman (1994) used the theory of belief functions in order to represent the uncertainty in knowledge parameters. Fuzzy rules use the concept of fuzzy logic (Mamdani & Gaines, 1981) to deal with imprecise

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knowledge. Ishibuchi, Nozaki, and Tanaka (1992) introduced the concept of distributed representation of fuzzy rules and applied it to classification problems. Wang and Mendel (1992) developed a method to generate fuzzy rules from numerical data. Lv, Zhu, and Tang (2007) presented a fuzzy classifier with probabilistic IF-THEN rules identified from training data.

However, all the previously mentioned methods suffer from a serious limitation: they fail to present a unified generic scheme to combine various types of uncertainties. This research takes advantage of recent research in the field of evidence combination under uncertainty, and introduces a generalised rule-based representation scheme which is capable of handling various types of uncertainty. The main contributions of the article are the extension of the original belief rule-base inference methodology using the evidential reasoning approach by i) introducing generalised belief rules as knowledge representation scheme, and ii) using the evidential reasoning rule for evidence combination in the rule-base inference methodology instead of the evidential reasoning approach.

Generalised belief rules, proposed in this article, are an extension of traditional IF-THEN rules where each consequent of a generalised belief rule is a belief distribution defined on the power set of hypotheses (propositions) that are assumed to be mutually exclusive and collectively exhaustive. While both are generalisations of traditional IF-THEN rules with distributed assessments in rule consequents, generalised belief rules are different from the belief rules introduced by Yang et al. (2006) The belief rules and the inference mechanism discussed in the work of Yang et al. (2006) are explicitly for rules with consequents that are belief distributions defined on a set of hypotheses itself. Whereas the belief rules and the inference mechanism investigated in this article are explicitly for rules with a consequent that is a belief distribution defined on the power set of the hypotheses.

This novel extension allows any combination of certain, uncertain, interval, or unknown judgements to be represented as rule-based knowledge. It is shown, in Section 3, that traditional IF-THEN rules, probabilistic rules, and interval rules are all special cases of the new generalised belief rules.

Expert knowledge under this framework is represented using attributes, hypotheses, and belief rules. Each attribute represents either an independent variable or a dependent variable related to a problem domain. The framework is able to handle both qualitative and quantitative attributes. A finite set of hypotheses, that are assumed to be mutually exclusive and collectively exhaustive, is defined for each attribute regardless of whether the attribute is qualitative or quantitative. The set of hypotheses is called the frame of discernment and it is usually denoted by Θ in the literature. Each element of the frame of discernment represents a hypothesis or a possible outcome of that attribute. Hypotheses are sometimes referred to in the literature as *instances* of a variable (Heckerman, 1992, 1993); *propositions* (MacKay, 2003; Yang et al., 2006); *grade values* (Yang, Wang, Xu, Chin, & Chatton, 2012); *assessment grades* (Yang & Xu, 2013); or *evaluation grades* (Si, Hu, Yang, & Zhou, 2011). For the sake of clarity, the term of hypothesis is hereinafter used to only refer to an element of the frame of discernment, the term of referential value is used to refer to an assignment of a numeric value to a hypothesis of an attribute, and the term of proposition is used to refer to a focal element of the power set of the frame of discernment. For example, a *Risk* attribute can be evaluated using three hypotheses: *Low, Medium*, or *High*. Three referential values can be assigned to the hypotheses of the *Risk* attribute in the previous example: *Low* = 0.0, *Medium* = 0.5, and *High*=1.0. A belief degree can be assigned to any of the eight elements of the power set of the frame of discernment ($\emptyset$, $\{Low\}$, $\{Medium\}$, $\{High\}$, $\{Low,\ Medium\}$, $\{Low,\ High\}$, $\{Medium,\ High\}$, $\{Low, Medium, High\}$). Belief degrees here refer to the use of probabilities to describe degrees of belief in propositions (MacKay, 2003).

The reminder of this article is summarised as follows. In the following section, we review the work done previously. The formal definition of the new knowledge representation scheme is given and discussed in Section 3. Section 4 describes how the original method for belief rule base inference has been extended to allow inference under the new knowledge representation scheme. Section 5 presents two illustrative examples of and briefly demonstrates the implementation of the new knowledge representation scheme and the new inference methodology in Generic Expert System. Finally, Section 6 presents our conclusions and improvements to be considered in future work.

2. Background

2.1. Belief rule base inference methodology using the evidential reasoning approach (RIMER)

The *belief rule base inference methodology using the evidential reasoning approach*, abbreviated as RIMER, was introduced by Yang et al. (2006) as a generic complex system modelling methodology under uncertainty. The general RIMER methodology consists of three components: (i) knowledge representation scheme; (ii) inference engine; and (iii) a learning model for belief rule base optimisation.

Knowledge representation scheme in RIMER

Knowledge and knowledge parameters are elicited from domain experts. Experts give their subjective judgements in a form of belief distributions representing the degree to which they believe a consequent is likely to happen given the conditions specified by the antecedent attributes. Eliciting knowledge representation parameters from experts for all rules in the rule base can be time-consuming as the number of such subjective judgements required is finite but very large.

In a traditional rule base, an IF-THEN rule is described as:

$$R : \text{IF } x_1 \text{ is } a_1 \wedge x_2 \text{ is } a_2 \wedge \cdots \wedge x_n \text{ is } a_n \text{ THEN } \ D \tag{1}$$

The consequent in the previous traditional IF-THEN rule, viz. D in Eq. (1), is believed to be either 100% true or 100% false. Such strict knowledge representation scheme leaves no room for uncertain or incomplete judgements and therefore cannot be applied to domains where uncertain or incomplete knowledge is involved.

To overcome this limitation, *belief rules* extend traditional IF-THEN rules by replacing single-valued consequents with a distributed assessment (Yang et al., 2006). It is realised that using distributed assessment instead of single numbers would enable various types of information to be incorporated into a decision making process without pre-aggregation (Zhang et al., 1989). According to Yang et al. (2006), a belief rule extended version of the rule in (Eq. (1)) is defined as:

$$\begin{aligned} R' :\quad & \text{IF } x_1 \text{ is } a_1 \ \wedge\ x_2 \text{ is } a_2 \ \wedge\ \ldots\ \wedge\ x_n \text{ is } a_n \\ & \text{THEN } \{(d_1,\beta_1),(d_2,\beta_2),\ldots(d_m,\beta_m)\}\beta_i \geq 0, \text{ and } \sum_{i=1}^{N}\beta_i \leq 1 \end{aligned} \tag{2}$$

In the previous belief rule, a distribution is used as a consequent instead of a single value. The distribution is called a *belief structure* (Yang et al., 2006). A belief structure is a distribution defined on a set of hypotheses that are assumed to be collectively exhaustive and mutually exclusive (Wang, Yang, & Xu, 2006). A belief degree is assigned to each hypothesis representing the degree to which the hypothesis is believed to be certain. Note that the sum of believe degrees assigned to all hypotheses in a rule consequent