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Chaotic time series prediction via artificial neural square fuzzy inference system

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ABSTRACT

The present article investigates the application of second order TSK (Takagi Sugeno Kang) fuzzy systems in predicting chaotic time series. A method has been introduced for training second order TSK fuzzy systems using ANFIS (Artificial Neural Fuzzy Inference System) training method. In a second order TSK system existence of nonlinear terms in the rules' consequence prohibits use of current available ANFIS codes as is but the proposed method makes it possible to use ANFIS for a class of simplified second order TSK systems. The main impact of this method on the expert and intelligent systems is to provide a new way for modeling and predicting the future situation of more complex phenomena with a smaller decision rule base. The most significance of the proposed method is the simplicity and available code reuse property. As a case study the proposed method is used for the prediction of chaotic time series. Error comparison shows that the proposed method trains the second order TSK system more effectively.

1. Introduction

TSK fuzzy systems are of the most popular fuzzy systems in modeling nonlinear functions (Takagi & Sugeno, 1985). The consequence of each fuzzy rule in these systems is a constant value or a linear combination of input variables that are called zero order and first order TSK fuzzy systems respectively. TSK fuzzy systems aim to simplify and speed-up defuzzification calculations, which is a bottle-neck in real time control systems (Takács, 2004; Jassbi, Serra, Ribeiro, & Donati, 2006). This simplification leads to loss of some key properties of native fuzzy systems (namely Mamdani Systems) (Takács, 2004; Hamam & Georganas, 2008).

Also using traditional TSK systems in modeling complex problems including multivariable nonlinear functions leads to fuzzy systems with many rules and membership functions (MFs) which is undesirable (Ren, 2007). High order TSK fuzzy systems have been introduced to overcome this problem and to increase the transparency and interpretability of TSK systems while preserving computational advantages (Ren, 2007; Buckley, 1992; Demirli & Muthukumaran, 2000; Kasabov, 2002; Cavallo, 2005; Herrera, Pomares, Rojas, Valenzuela, & Prieto, 2005).

Prediction of chaotic time series (specially the Mackey–Glass time series as a benchmark (Mackey & Glass, 1977)) is regarded as a popular research field of the applied mathematics (e.g. (Kasabov, 2002; Kalhor, Araabi, & Lucasi, 2010; Almaraashi & Coupland, 2010; Almaraashi & John, 2011; Sugiarto & Natarajan, 2007; Chen, Yang, Dong, & Abraham, 2005)). TSK fuzzy systems are one of the most useful tools in predicting chaotic time series (Kasabov, 2002). Using high order TSK fuzzy systems in the prediction of chaotic time series has received more attention in recent years (Askari & Montazerin, 2015; Gangwar & Kumar, 2012; Chen & Chen, 2011; Chen & Tanuwijaya, 2011; Egrioglu, Aladag, Yolcu, Uslu, & Basaran, 2010; Kalhor et al., 2010; Kasabov, 2002). Some other applications of high order TSK systems in this field can be found by Theochaires, 2006; Song, Ma, and Kasabov, 2005; Kim, Kyung, Park, Kim, and Park, 2004). High order TSK systems will be much more useful if one finds a good training method for them similar to ANFIS for zero and first order TSK systems (Jang, 1993). In this paper first the structure of a second order TSK system is explained and a Transformed Second order TSK Learning method is proposed to tune parameters of a second order TSK system as it was possible to train zero and first order TSK systems by available ANFIS codes.

2. Second order TSK system modification

2.1. The approximation property of high order TSK systems

According to Wang, 1997 (Section 9.2) all fuzzy systems defined over the compact n-dimensional space D that:

- Use Gaussian input membership functions.

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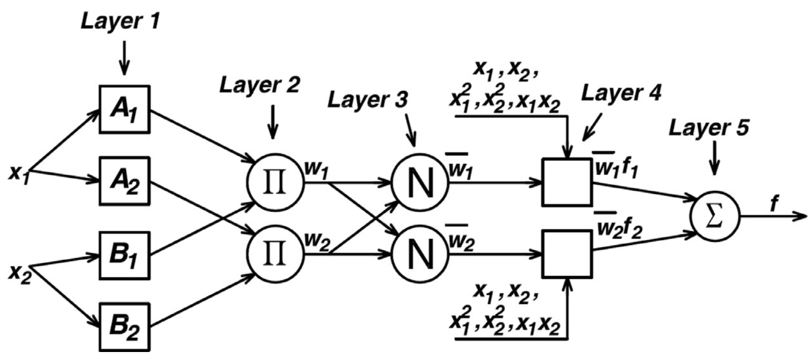

Fig. 1. Second order TSK system as the Type-3 fuzzy reasoning system.

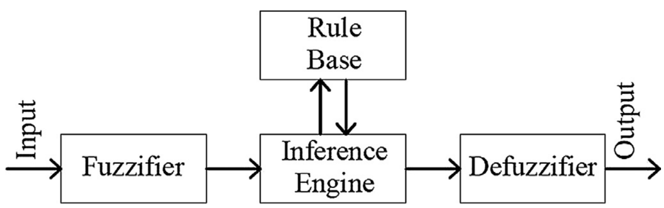

Fig. 2. Structure of a fuzzy system.

- Employ production for the AND method (t-norm) and aggregation.
- Uses center average as the deffuzification method.

fulfill the Stone–Weierstrass conditions for any continuous function like $g(x)$ defined on D, namely if S represents the set of all possible fuzzy systems defined on D while preserving the above specifications, then for any $g \in C(D)$ and any real number like $\varepsilon > 0$ there exists a fuzzy system $F \in S$ that can approximate g with an absolute error less than ε. If $F(x)$ represents the fuzzy system output for input vector $x \in D$ then:

$$\forall \varepsilon > 0 \,\&\, \forall g \in C(D) \exists\, F \in S\, s.t. |F(x) - g(x)| < \varepsilon\, \forall x \in D \tag{1}$$

So such fuzzy systems can perform nonlinear function approximation with an arbitrary precision. Also Jang, 1993 claims that TSK fuzzy system with bell shaped input membership function can perform similar approximation too. As the view point of (Jang, 1993) a high order TSK system is a type-3 fuzzy reasoning one, which also uses higher degrees of input monomials in its layer 4 (as illustrated in Fig. 1 for a second order TSK system) (Heydari, Gharaveisi, & Vali, 2015).

Considering the same proof as mentioned by Jang, 1993, and the fact that the "Simplified ANFIS" by Jang, 1993 is also a proper subset of high order TSK systems it can be concluded that high order TSK systems are also universal approximators (Heydari et al., 2015).

As a managerial insight, prediction is a key factor in planning. High order TSK systems are also powerful tools for performing prediction. For example precise electricity (Eslahi Tatafi, Heydari, & Gharaveisi, 2014), oil or gold (Rajaee, 2013) price prediction is too valuable for pricing a factory product or to adjust budgetary for future.

2.2. General second order TSK system

Considering the general structure of a fuzzy system as Fig. 2, the rule base is the most different block in a second order TSK system.

Supposing a two input TSK system, the lth rule will be:

$$y^l = a^l_{00} + a^l_{10}x_1 + a^l_{20}x_2 + a^l_{11}x_1^2 + a^l_{22}x_2^2 + a^l_{12}x_1x_2 \tag{2}$$

And the output will be the weighted average of rules' outputs:

$$y = \frac{\sum_{l=1}^{r} w^l y^l}{\sum_{l=1}^{r} y^l} \tag{3}$$

where x_1 and x_2 are input variables, r is the total number of rules and w^l is firing strength of each rule determined by input membership function and the fuzzification block.

As mentioned by Jang, 1993 the training method is a key factor in the approximation quality of TSK systems, however a disappointing fact about high order TSK systems is that a straightforward learning method the same as ANFIS cannot be found in the literature that provides suitable speed and precision together (Herrera et al., 2005). For example Kasabov, 2002 replaces the consequence function by a small neural network to achieve high order TSK, also Kalhor et al., 2010 use deformed linear models which involves sophisticated optimization for finding optimum deformed linear models or a sequential learning method has been developed by Heydari et al., 2015. A suitable training process must tune input membership functions and determine values of rules' coefficients (e.g., $a^1{}_{00}$,$a^1{}_{11}$,...) such that the overall fuzzy system performs desirably.

According to Jang, 1993 and software applications available for tuning TSK systems parameters (like MATLAB fuzzy logic toolbox) it is seen that current codes do not support polynomials of a degree higher than one, as the consequent part of a rule in a TSK fuzzy inference system. While high order TSK systems include polynomials of degrees higher than one, as a contribution we introduce a transformation method for a class of second order TSK systems, by which it is possible to use the traditional ANFIS for tuning the parameters of these second order TSK systems.

2.3. Square TSK system (S-TSK) (Heydari, 2015)

Considering a simplified second order TSK system by omitting terms which consist of more than one variable (like x_1x_2 in (2)), the typical form of the consequent of a rule say in a two input system will be:

$$y^l = y^l = a^l_{00} + a^l_{10}x_1 + a^l_{20}x_2 + a^l_{11}x_1^2 + a^l_{22}x_2^2 \tag{4}$$

Now one might consider:

$$\begin{aligned} z_1 &= x_1^2 \\ z_2 &= x_2^2 \end{aligned} \tag{5}$$

And we get a new first order TSK system with four variables: x_1,x_2,z_1 and z_2, which its lth rule's consequent will be:

$$y^l = a^l_{00} + a^l_{10}x_1 + a^l_{20}x_2 + a^l_{11}z_1 + a^l_{22}z_2 \tag{6}$$

where the parameters $a^l_{00}, a^l_{10}, a^l_{20}, a^l_{11}$ and a^l_{22} can be found by ANFIS. This is the idea behind defining such simplified second order TSK system called in this paper a Square TSK (or S-TSK)