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Early online detection of high volatility clusters using Particle Filters

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ABSTRACT

This work presents a novel online early detector of high-volatility clusters based on uGARCH models (a variation of the GARCH model), risk-sensitive particle-filtering-based estimators, and hypothesis testing procedures. The proposed detector utilizes Risk-Sensitive Particle Filters (RSPF) to generate an estimate of the volatility probability density function (PDF) that offers better resolution in the areas of the state-space that are associated with the incipient appearance of high-volatility clusters. This is achieved using the Generalized Pareto Distribution for the generation of particles. Risk-sensitive estimates are used by a detector that evaluates changes between prior and posterior probability densities via asymmetric hypothesis tests, allowing early detection of sudden volatility increments (typically associated with early stages of high-volatility clusters). Performance of the proposed approach is compared to other implementations based on the classic Particle Filter, in terms of its capability to track regions of the state-space associated to a greater financial risk. The proposed volatility cluster detection scheme is tested and validated using both simulated and actual IBM's daily stock market data.

1. Introduction

Volatility of returns is a well-studied variable in Finance, mainly because its relevance in pricing and risk management. Since the work of Mandelbrot in the 1960's, it has been widely accepted that volatility presents itself in temporal clusters, where large price variations are followed by large variations (Cont, 2005; Mandelbrot, 1963). Multiple researchers have tried to model the complex behavior of volatility, being the GARCH model (Bollerslev, 1986) the first to capture these temporal cluster properties. The wide recognition for the GARCH models has given rise to a whole family of structures, in which stochastic variations have been lately introduced.

On the one hand, from an engineering perspective, early online detection of high-volatility clusters in a stochastic environment poses an interesting problem, since detection algorithms must be designed to monitor a latent (non-observable) state; simultaneously tracking disturbances introduced by other non-measurable variables that are always present in complex systems (such as stock markets). In fact, from the standpoint of state-space modeling for financial time series, volatility is a non-observable state, while continuously compounded returns can be associated with daily measurements. Given that inference on financial volatility is necessary to detect high risk events, the challenge is then to propose detection frameworks based on accurate and precise estimates of this non-observable state.

On the other hand, in Finance, the words "early online detection" have reached unsuspected relevance in a world where is now possible to use information from high-volatility cluster detectors (which could have been originally focused on very specific and critical markets) for the implementation of online predictive strategies at a global-scale. Consider, for example, the implementation of intelligent expert systems that could recommend optimal corrective actions for Latin American markets based on online anomaly detectors analyzing Asian markets during the early morning. Indeed, the development of tools for online early detection of high-volatility clusters (such as the one proposed in this article) generates appropriate conditions for the implementation of novel online schemes for optimal decision-making in Finance; a task where the whole community working on expert and intelligent systems may contribute in the near future.

These fundamental questions have motivated in recent years substantial research with focus in the detection of structural breaks (or model parameter changes) in financial variables, with the purpose of understanding market shocks or anomalies (Chan & Koop, 2014; Chen, Gerlach, & Liu, 2011; He & Maheu, 2010; Rapach & Strauss, 2008; Ross, 2013). Given the complexities involved in modeling volatility, several approaches have been proposed, including new models such as the structural break GARCH (SB-GARCH). For these models that include stochastic volatility and

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breaks, the most common approach for the estimation of volatility has been sequential Monte Carlo methods (a.k.a. Particle Filters) (Arulampalam, Maskell, Gordon, & Clapp, 2002; Doucet, Godsill, & Andrieu, 2000), because of its good performance, flexibility, and the possibility to estimate model parameters online (Liu and West, 2001, chap. 10). Further efforts have been spent in the study of jumps (or discontinuities) of returns and volatility (Andersen, Tim, & Diebold, 2007; Eraker, Johannes, & Polson, 2003; Laurent, Lecourt, & Palm, 2014; Lee & Mykland, 2008), although these approaches impose restrictions for the quality of data that may be difficult to address. In fact, most of the available tests for detection of jumps include high-frequency, intra-day information of the studied variables or high liquidity of the assets (Laurent et al., 2014), or offline tests.

In this regard, we propose a novel online early detector of high-volatility clusters based on unobserved GARCH models (Tobar, 2010) (uGARCH, a variation of the GARCH model), Risk-Sensitive Particle-Filters (RSPF) estimators (Thrun, Langford, & Verma, 2002), and hypothesis testing procedures. The proposed detector utilizes risk-sensitive particle filters to generate an estimate of the volatility probability density function (PDF) that offers better resolution in the areas of the state-space that are associated with the incipient appearance of high-volatility clusters. Risk-sensitive estimates are used by a detector that evaluates changes between prior and posterior probability densities via asymmetric hypothesis tests, allowing early detection of sudden volatility increments (typically associated with early stages of high-volatility clusters). This algorithm is tested in simulated data (where volatility is known), as well as IBM's stock market data, where volatility has to be estimated (since ground truth cannot be measured). To the best of our knowledge, this is the first attempt in financial econometrics to perform online detection of events by contrasting the information that is present in priors and posterior probability densities estimates in Bayesian estimation frameworks.

The main contributions of this article are:

- Implementation and assessment of a novel method for the generation of volatility estimators, based on RSPF, that provides better resolution in the areas of the state-space associated with the appearance of high-volatility clusters.
- Implementation and assessment of early detection schemes for high-volatility clusters based on the comparison between prior and posterior particle-filtering-based estimates.
- A throughout performance comparison between RSPF and classic sequential Monte Carlo methods in terms of their effectiveness when used in early detection of high-volatility clusters.

The structure of this article is as follows. Section 2 presents a literature review on the use of Bayesian frameworks for Financial volatility estimation. Section 3 presents the proposed method for early detection of high-volatility clusters. Section 4 presents performance measures to be used in the assessment of obtained results, provides a sensitivity analysis for the proposed method using simulated data (where the ground truth value of the unmeasured state is known), and finalizes with a throughout performance analysis for the proposed method based on actual IBM stock data. Section 5 presents a few interesting general remarks, while Section 6 shows the main conclusions related to this research.

2. A Bayesian framework for volatility estimation

Monte Carlo (MC) and *Markov Chain Monte Carlo* (MCMC) methods have been widely used to approximate integrals and probability density functions (Tobar, 2010). Nevertheless, their use in Bayesian inference is not direct, since this problem involves a sequence of time-variant probability density functions; while MCMC assumes that the objective density is time-invariant. This prompted the development of a sequential version of Monte Carlo integration, one that is able to use measurements to improve recursive estimation.

The first section of this section introduces the uGARCH model, a stochastic volatility model based on the well-known GARCH(1,1) model (Bollerslev, 1986). Then, the tracking problem is presented in Section 2.2, providing insight about the problems encountered in a Bayesian filtering framework. Also, Monte Carlo integration and the *importance sampling* method are presented. This opens the possibility to explore the Particle Filter and the Risk Sensitive Particle Filter, which may be employed in a stochastic volatility estimation framework. Finally, Section 2.3 explains the need for online parameter estimation.

2.1. The uGARCH model

The uGARCH model can be seen as a state-space structure that allows the implementation of a Bayesian framework for the purpose of volatility estimation.

The uGARCH model (Tobar, 2010) assumes that the dynamics of volatility are not driven by the observed process $u_k = r_k - \mu_{k|k-1}$. Instead, they are driven by a non-observable process u'_k which has the same distribution as u_k. The uGARCH model is defined as:

$$\sigma_k^2 = \omega + \alpha\sigma_{k-1}^2\eta_k^2 + \beta\sigma_{k-1}^2, \tag{1}$$

$$r_k = \mu + \sigma_k\epsilon_k, \tag{2}$$

where r_k is a process of returns, σ_k is the stochastic volatility, $\mu \in \mathbb{R}^+$, $\omega \in \mathbb{R}^+$, and α, $\beta > 0$ are parameters, with $\alpha + \beta < 1$. Furthermore, $\epsilon_k \sim \mathcal{N}(0, 1)$ and $\eta_k \sim \mathcal{N}(0, \sigma_\eta^2)$ are *i.i.d.*[1] processes for every time step k.

It is necessary to note from Eqs. (1) and (2) that the subscripts are not written conditionally: at time step k, σ_k^2 is not known without uncertainty, given Σ_{k-1}.

To completely define the model, it is necessary to present the state transition distribution $p(\sigma_k^2|\sigma_{k-1}^2)$ and the likelihood $p(r_k|\sigma_k^2)$:

$$p(\sigma_k^2|\sigma_{k-1}^2) = \frac{1}{\sqrt{2\pi\sigma_\eta^2\alpha\sigma_{k-1}^2\left(\sigma_k^2 - \omega + \beta\sigma_{k-1}^2\right)}} \cdot \exp\left[\frac{\sigma_k^2 - \omega + \beta\sigma_{k-1}^2}{2\sigma_\eta^2\alpha\sigma_k^2}\right], \qquad \sigma_k^2 \geq \omega + \beta\sigma_{k-1}^2. \tag{3}$$

$$p(r_k|\sigma_k^2) = \frac{1}{\sqrt{2\pi\sigma_k^2}}\exp\left(-\frac{(r_k-\mu)^2}{2\sigma_k^2}\right). \tag{4}$$

For the complete derivation of Eq. (4), please refer to Mundnich (2013). The calculation and presentation of Eq. (4) makes the use of a generic Particle Filtering approach for volatility estimation in this model possible.

2.2. The Particle Filter

State-space models consider a transition equation that describes the prior distribution of a hidden Markov process $\{x_k; k \in \mathbb{N}\}$, called the state process, and an observation equation describing the likelihood of the observation $\{z_k; k \in \mathbb{N}\}$ (Doucet et al., 2000):

$$x_k = f(x_{k-1}, \upsilon_{k-1}), \tag{5}$$

$$z_k = h(x_k, w_k), \tag{6}$$

where $f(\cdot,\ \cdot)$ is a state-transition function with corresponding $\{\upsilon_{k-1}, k \in \mathbb{N}\}$ *i.i.d.* innovation process, and $h(\cdot,\ \cdot)$ is the observation

[1] Independent and identically distributed.