A new approach for ranking of *L–R* type generalized fuzzy numbers

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ABSTRACT

Ranking of fuzzy numbers play an important role in decision making, optimization, forecasting etc. Fuzzy numbers must be ranked before an action is taken by a decision maker. Cheng (Cheng, C. H. (1998). A new approach for ranking fuzzy numbers by distance method. Fuzzy Sets and Systems, 95, 307–317) pointed out that the proof of the statement "Ranking of generalized fuzzy numbers does not depend upon the height of fuzzy numbers" stated by Liou and Wang (Liou, T. S., & Wang, M. J. (1992). Ranking fuzzy numbers with integral value. Fuzzy Sets and Systems, 50, 247–255) is incorrect. In this paper, by giving an alternative proof it is proved that the above statement is correct. Also with the help of several counter examples it is proved that ranking method proposed by Chen and Chen (Chen, S. M., & Chen, J. H. (2009). Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Expert Systems with Applications, 36, 6833–6842) is incorrect. The main aim of this paper is to modify the Liou and Wang approach for the ranking of *L–R* type generalized fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. It is shown that proposed ranking function satisfy all the reasonable properties of fuzzy quantities proposed by Wang and Kerre (Wang, X., & Kerre, E. E. (2001). Reasonable properties for the ordering of fuzzy quantities (I). Fuzzy Sets and Systems, 118, 375–385).

1. Introduction

Fuzzy set theory (Zadeh, 1965) is a powerful tool to deal with real life situations. Real numbers can be linearly ordered by $\geqslant$ or $\leqslant$, however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision-making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory.

The method for ranking was first proposed by Jain (1976). Yager (1981) proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0, 1]. In Kaufmann and Gupta (1988), an approach is presented for the ranking of fuzzy numbers. Campos and Gonzalez (1989) proposed a subjective approach for

ranking fuzzy numbers. Liou and Wang (1992) developed a ranking method based on integral value index. Cheng (1998) presented a method for ranking fuzzy numbers by using the distance method. Kwang and Lee (1999) considered the overall possibility distributions of fuzzy numbers in their evaluations and proposed a ranking method. Modarres and Sadi-Nezhad (2001) proposed a ranking method based on preference function which measures the fuzzy numbers point by point and at each point the most preferred number is identified. Chu and Tsao (2002) proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu (2005) presented a centroid-index method for ranking fuzzy numbers. Liang, Wu, and Zhang (2006) and Wang and Lee (2008) also used the centroid concept in developing their ranking index. Chen and Chen (2007) presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari (2009) introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α-levels of trapezoidal fuzzy numbers. Chen and Chen (2009) presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads.

In this paper, with the help of several counter examples it is shown that ranking method proposed by Chen and Chen (2009) is incorrect. The main aim of this paper is to propose a new approach for the ranking of *L–R* type generalized fuzzy numbers.

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It is shown that the ranking of L–R type generalized fuzzy numbers does not depend upon the height of fuzzy number. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

This paper is organized as follows: In Section 2, some basic definitions, arithmetic operations and ranking function are reviewed. In Section 3, the shortcomings of Chen and Chen (2009) approach is discussed. In Section 4, a new approach is proposed for the ranking of L–R type generalized fuzzy numbers. In Section 5, the correct ordering of fuzzy numbers are obtained and also it is shown that the proposed approach satisfies all the reasonable properties of fuzzy quantities. The conclusion is discussed in Section 6.

2. Preliminaries

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

2.1. Basic definitions

Definition 1 (Kaufmann and Gupta, 1988). The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\widetilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\widetilde{A}} : X \rightarrow [0, 1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\widetilde{A}}$ is called the membership function and the set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$ defined by $\mu_{\widetilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2 (Kaufmann and Gupta, 1988). A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\widetilde{A}} : R \rightarrow [0, 1]$ is continuous.
2. $\mu_{\widetilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\widetilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\widetilde{A}}(x) = 1$ for all $x \in [b,c]$, where $a < b < c < d$.

Definition 3 (Kaufmann and Gupta, 1988). A fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a < x < b, \\ 1, & b \leqslant x \leqslant c, \\ \frac{(x-d)}{(c-d)}, & c < x < d. \end{cases}$$

Definition 4 (Chen and Chen, 2009). A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\widetilde{A}} : R \rightarrow [0, w]$ is continuous.
2. $\mu_{\widetilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\widetilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\widetilde{A}}(x) = w$, for all $x \in [b,c]$, where $0 < w \leqslant 1$.

Definition 5 (Dubois and Prade, 1980). A fuzzy number $\widetilde{A} = (a, b, c, d; w)_{LR}$ is said to be a L–R type generalized fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} wL\left(\frac{b-x}{b-a}\right), & \text{for } a < x < b, \\ w, & \text{for } b \leqslant x \leqslant c, \\ wR\left(\frac{x-c}{d-c}\right), & \text{for } c < x < d. \end{cases}$$

where L and R are reference functions.

Definition 6 (Dubois and Prade, 1980). A L–R type generalized fuzzy number $\widetilde{A} = (a, b, c, d; w)_{LR}$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} w\frac{(x-a)}{(b-a)}, & a < x < b, \\ w, & b \leqslant x \leqslant c, \\ w\frac{(x-d)}{(c-d)}, & c < x < d. \end{cases}$$

2.2. Arithmetic operations

In this subsection, arithmetic operations between two L–R type generalized fuzzy numbers, defined on universal set of real numbers R, are reviewed (Dubois & Prade, 1980).

Let $\widetilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)_{LR}$, $\widetilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)_{LR}$ and $\widetilde{A}_3 = (a_3, b_3, c_3, d_3; w_3)_{RL}$ be any L–R type generalized fuzzy numbers and then

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; min(w_1, w_2))_{LR}$
(ii) $\widetilde{A}_1 \ominus \widetilde{A}_3 = (a_1 - d_3, b_1 - c_3, c_1 - b_3, d_1 - a_3; min(w_1, w_3))_{LR}$
(iii) $\lambda\widetilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1)_{LR} & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1)_{RL} & \lambda < 0. \end{cases}$

2.3. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function (Jain, 1976) $\Re : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $\widetilde{A} \succ \widetilde{B}$ iff $\Re(\widetilde{A}) > \Re(\widetilde{B})$
(ii) $\widetilde{A} \prec \widetilde{B}$ iff $\Re(\widetilde{A}) < \Re(\widetilde{B})$
(iii) $\widetilde{A} \sim \widetilde{B}$ iff $\Re(\widetilde{A}) = \Re(\widetilde{B})$

Remark 1 (Wang and Kerre, 2001). For all fuzzy numbers $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$ and $\widetilde{D}$ we have

(i) $\widetilde{A} \succ \widetilde{B} \Rightarrow \widetilde{A} \oplus \widetilde{C} \succ \widetilde{B} \oplus \widetilde{C}$
(ii) $\widetilde{A} \succ \widetilde{B} \Rightarrow \widetilde{A} \ominus \widetilde{C} \succ \widetilde{B} \ominus \widetilde{C}$
(iii) $\widetilde{A} \sim \widetilde{B} \Rightarrow \widetilde{A} \oplus \widetilde{C} \sim \widetilde{B} \oplus \widetilde{C}$
(iv) $\widetilde{A} \succ \widetilde{B}$, $\widetilde{C} \succ \widetilde{D} \Rightarrow \widetilde{A} \oplus \widetilde{C} \succ \widetilde{B} \oplus \widetilde{D}$

3. Shortcomings of Chen and Chen (2009) approach

In this section, the shortcomings of Chen and Chen approach on the basis of reasonable properties of fuzzy quantities and on the basis of height of fuzzy numbers, are pointed out.

3.1. On the basis of reasonable properties of fuzzy quantities (Wang & Kerre, 2001)

Let $\widetilde{A}$ and $\widetilde{B}$ be any two fuzzy numbers then
$\widetilde{A} \succ \widetilde{B} \Rightarrow \widetilde{A} \ominus \widetilde{B} \succ \widetilde{B} \ominus \widetilde{B}$ (Using remark 1)
i.e., $\Re(\widetilde{A}) > \Re(\widetilde{B}) \Rightarrow \Re(\widetilde{A} \ominus \widetilde{B}) > \Re(\widetilde{B} \ominus \widetilde{B})$
In this subsection, several examples are chosen to prove that the ranking function, proposed by Chen and Chen, does not satisfy the reasonable property, $\widetilde{A} \succ \widetilde{B} \Rightarrow \widetilde{A} \ominus \widetilde{B} \succ \widetilde{B} \ominus \widetilde{B}$, for the ordering