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Set-valued pseudo-metric families and Ekeland's variational principles in fuzzy metric spaces ☆

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Abstract

In this paper, we introduce a set-valued pseudo-metric family on a fuzzy metric space and the notion of compatibility between the set-valued pseudo-metric family and the original fuzzy metric. By means of this notion, we prove a general set-valued EVP, where the perturbation involves a set-valued pseudo-metric family compatible with the original fuzzy metric. From the general EVP, we deduce several particular EVPs, which extend the EVPs in Qiu (2013) [36] and in Gutiérrez et al. (2008) [20] to fuzzy metric spaces. By using set-valued pseudo-metric families and using the unified approach for approximate solutions introduced by Gutiérrez, Jiménez and Novo, we deduce a general version of set-valued EVP based on (C, ϵ)-efficient solutions in fuzzy metric spaces, where C is a coradiant set contained in the order cone. By choosing two specific versions of the coradiant set C in the general version of EVP, we obtain several particular set-valued EVPs for ϵ-efficient solutions in the sense of Németh and of Dentcheva and Helbig, respectively. These EVPs improve and generalize the related known results.
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1. Introduction

In 1972, Ekeland (see [12–14]) gave a variational principle, now known as Ekeland variational principle (briefly, denoted by EVP), which says that for any lower semi-continuous function f bounded from below on a complete metric space, there exists a slightly perturbed version of this function that has a strict minimum. In the last four decades, the famous EVP emerged as one of the most important results of nonlinear analysis and it has significant applications in optimization, optimal control theory, game theory, fixed point theory, nonlinear equations, dynamical systems, etc; see, for example, [6,13,14,17,21]. Motivated by this wide usefulness, many authors have been interested

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in extending EVP to vector-valued maps with values in a pre-ordered vector space, see, for example, [1,4–8,17,18,20,21,31,35,36,39,41,42,45] and the references therein. Recently, Gutiérrez, Jiménez and Novo [20] introduced a set-valued metric on a metric space. By using it they gave an original approach to extending the well-known scalar EVP to vector-valued maps. The new EVP they obtained is stronger than the previous EVPs from two points of view. First, because a set-valued perturbed map is considered, and second, because, in a certain sense, it does not depend on any ϵ-efficiency concept; for ϵ-efficiency, see the following Section 3. They also deduced several special versions of EVP involving approximate solutions for vector optimization problems and discussed their applications. However, in their work the assumption that the order cone D is w-normal is required (see [20]). Concerning w-normal cones, see the following Section 3 or [17,20,36]. This requirement restricts the applicable extent of the new versions of EVP. Qiu [36] considered a slightly more general notion: set-valued quasi-metrics and introduced the notion of compatibility between a set-valued quasi-metric and the original metric. By means of this notion, he proved a general set-valued EVP, where the perturbation contains a set-valued quasi-metric which is compatible with the original metric. Here, one needs not assume that the order cone is w-normal. From the general set-valued EVP, Qiu deduced a number of particular set-valued EVPs for approximate solutions in vector optimization problems, which improve and generalize the related results in [20].

On the other hand, we remark that in the above Gutiérrez, Jiménez and Novo's work and Qiu's work, the framework is a metric space. However, in many situations, the distance between two points is inexact rather than a single real number. When the uncertainty is due to fuzziness, as sometimes in the measurement of an ordinary length, it seems that the concept of a fuzzy metric space is more suitable. In 1984, Kaleva and Seikkala [27] introduced the concept of fuzzy metric spaces by setting the distance between two points to be a nonnegative fuzzy real number. From then on, a number of important results for maps on fuzzy metric spaces, such as variational principles, coincidence theorems and fixed point theorems, etc., were obtained in subsequent works (see, e.g., [2,15,21,22,25,26,43–45]). Recently, Qiu [38] established a general set-valued EVP, where the objective function is a set-valued map defined on a fuzzy metric space and taking values in a partially ordered locally convex space and the perturbation involves a convex subset of the order cone and a quasi-metric family generating the fuzzy topology of the domain space.

Inspired by the above works, in this paper, we try to extend the main results in [36] from metric spaces to fuzzy metric spaces. First, instead of a set-valued quasi-metric on a metric space (see [20,36]), we introduce a set-valued pseudo-metric family on a fuzzy metric space and discuss the notion of compatibility between the set-valued pseudo-metric family and the original fuzzy metric. By means of this notion we prove a general set-valued EVP, where the objective function is a vector-valued map defined on a fuzzy metric space and taking values in a partially ordered locally convex space, and the perturbation involves a set-valued pseudo-metric family compatible with the original fuzzy metric. As in [36], here we still needn't assume that the order cone is w-normal. From the general EVP, we deduce a number of particular versions of EVP, which extend the EVPs in [20,36] to fuzzy metric spaces. By using set-valued pseudo-metric families and using the unified approach for approximate solutions introduced by Gutiérrez, Jiménez and Novo (see [19]), we deduce a general version of set-valued EVP based on (C, ϵ)-efficient solutions in fuzzy metric spaces, where $C \subset D\backslash\{0\}$ is a coradiant set. As shown in [19,20], by choosing different coradiant set C, we can obtain various ϵ-efficiency concepts in vector optimization. In particular, we deduce several special set-valued EVPs for ϵ-efficient solutions in the sense of Németh and in the sense of Dentcheva and Helbig, respectively, which extend the related results in [20,36]. Thus, in the framework of fuzzy metric spaces, we develop the theory of EVPs in vector optimization. Moreover, as we shall see, our EVPs also generalize and improve the related results in [1,4–6,17,18,21,22,25,26,38,42].

This paper is organized as follows. In Section 2, we recall some basic concepts and results on fuzzy numbers and fuzzy metric spaces. Section 3 presents some basic facts on partially ordered locally convex spaces and states some notions on efficient solutions and approximate solutions for vector optimization. In Section 4, we introduce a set-valued pseudo-metric family and the notion of compatibility between a set-valued pseudo-metric family and the original fuzzy metric. Then, we prove a general set-valued EVP, where the perturbation contains a set-valued pseudo-metric family compatible with the original fuzzy metric. From this, we deduce several particular versions of set-valued EVP. In Section 5, in the setting of fuzzy metric spaces, we give a version of set-valued EVP based on (C, ϵ)-efficient solutions. In Section 6, we obtain some conditions to fulfill the assumptions of these set-valued EVPs. In Sections 7 and 8, we consider set-valued EVPs for ϵ-efficient solutions in the sense of Németh and of Dentcheva and Helbig, respectively. Finally, by summarizing the results obtained and proposing further research in this direction, we present our conclusions in Section 9.

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