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An addendum to "Migrative uninorms and nullnorms over t-norms and t-conorms" ☆

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Abstract

The aim of this paper is to complete some results in the paper "Migrative uninorms and nullnorms over t-norms and t-conorms" [1]. That paper studied the alpha-migrativity of uninorms over t-norms. Here, migrativity in the other direction is investigated.

1. Introduction

As is pointed out by Fodor and Rudas [2], the migrativity of a t-norm T over a t-norm T_0 is equivalent to the migrativity of a t-norm T_0 over a t-norm T. Recently, Mas et al. studied the α-migrativity of uninorms over t-norms. But the other direction, i.e., α-migrativity of t-norms and t-conorms over uninorms is missing. However, we find that the migrativity of a uninorm U over a t-norm T is not equivalent to the migrativity of a t-norm T over a uninorm U. In this paper, we point out the similarities and differences between the migrativity of a uninorm U over a t-norm T and the migrativity of a t-norm T over a uninorm U and we do an analogous study for the migrativity of t-conorms over uninorms (see Sec. 3 for details). The case for t-norms/t-conorms over nullnorms will be handled similarly (see Sec. 4 for details).

2. Preliminaries

The basic notions and results of t-norms and t-conorms can be found in [3]. We will just give in this section some basic facts about uninorms.

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Definition 2.1. (See [3].) A binary operation $U : [0, 1]^2 \to [0, 1]$ is called a uninorm if it is associative, commutative, non-decreasing in each place and has a neutral element $e \in [0, 1]$.

A uninorm with neutral element $e = 1$ is clearly a t-norm and a uninorm with neutral element $e = 0$ is a t-conorm. Any uninorm U satisfies that $U(0, 1) \in \{0, 1\}$ and it is called conjunctive when $U(1, 0) = 0$ and disjunctive when $U(1, 0) = 1$.

Definition 2.2. (See Mas et al. [1].) Let U be a conjunctive uninorm. We will say that U is locally internal on the boundary if it satisfies $U(1, x) \in \{1, x\}$ for all $x \in [0, 1]$.

Similarly, if U is a disjunctive uninorm. We will say that U is locally internal on the boundary if it satisfies $U(0, x) \in \{0, x\}$ for all $x \in [0, 1]$.

Definition 2.3. (See [3].) A binary operation $F : [0, 1]^2 \to [0, 1]$ is called a nullnorm if it is associative, commutative, non-decreasing in each place and there exists $k \in [0, 1]$ called absorbing element that verifies $F(k, x) = k$ for all $x \in [0, 1]$ and $F(0, x) = x$ for all $x \le k$ and $F(1, x) = x$ for all $x \ge k$.

In that case, when $k = 0$ we obtain a t-norm and when $k = 1$ we obtain a t-conorm.

3. Migrativity of t-norms/t-conorms over uninorms

Definition 3.1. Given a uninorm U and $\alpha \in [0, 1]$. A t-norm T is said to be α-migrative over U or (α, U)-migrative if

$$T(U(\alpha, x), y) = T(x, U(\alpha, y)) \tag{1}$$

for all $x, y \in [0, 1]$.

Since the cases of t-norms and t-conorms are already known, we will consider only uninorms with neutral element $e \in]0, 1[$.

Remark 3.1. In [2], Fodor and Rudas pointed out that T is α-migrative with respect to T_0 if and only if T_0 is α-migrative with respect to T if and only if $T(\alpha, x) = T_0(\alpha, x)$ for all $x \in [0, 1]$. But, it may not be true when one of two t-norms is replaced by a uninorm. Below, we give some counterexamples.

Example 3.1. Let $T_M(x, y) = \min(x, y)$ for all $x, y \in [0, 1]$ be a t-norm and

$$U(x, y) = \begin{cases} \min(x, y) & \text{if } x, y \in [0, \frac{1}{2}], \\ \max(x, y) & \text{otherwise,} \end{cases}$$

be a uninorm with neutral element $e = \frac{1}{2}$. Then U is $(1, T_M)$-migrative (see [4] in detail). But,

$$T_M\Big(U\Big(1, \frac{1}{4}\Big), \frac{1}{3}\Big) = \frac{1}{3} \neq \frac{1}{4} = T_M\Big(\frac{1}{4}, U\Big(1, \frac{1}{3}\Big)\Big),$$

i.e., T_M is not $(1, U)$-migrative.

Example 3.2. For T_M and U in Example 3.1, routine calculation shows that T_M is $(\frac{1}{2}, U)$-migrative. But,

$$U\Big(T_M\Big(\frac{1}{2}, 0\Big), 1\Big) = 1 \neq 0 = U\Big(0, T_M\Big(1, \frac{1}{2}\Big)\Big),$$

i.e., U is not $(\frac{1}{2}, T_M)$-migrative.

Below, we analyze some of initial properties of (α, U)-migrative t-norms.