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Short communication

Notes on "Exact calculations of extended logical operations on fuzzy truth values"

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Abstract

In this note, we show by counterexamples that Theorems 9 and 10, and Propositions 7, 18, 19 and 20 in a previous paper by Gera and Dombi (2008) [1] contain some flaws and then we provide the correct versions.

Keywords: Fuzzy truth value; Extension principle; Type-2 fuzzy sets

1. Introduction

In order to strengthen the capability of modeling and manipulating inexact information in a logical manner, the concept of type-2 fuzzy sets was introduced by Zadeh [11]. In recent years, type-2 fuzzy sets have been utilized in many areas [2,3,5–9]. These applications give motivation to investigate the operation of type-2 fuzzy sets. Gera and Dombi provided some computationally formulas for extended t-norms and t-conorms [1]. However, we note that Theorems 9 and 10, and Propositions 17, 18, 19 and 20 are correct only if the t-norm is left-continuous. In this note, we represent some examples showing that they are no longer valid for non-continuous t-norms, and then we provide the correct versions.

2. Preliminary

In this section we will briefly recall the concepts of t-norms, t-conorms, implications and coimplications on the unit interval [0, 1].

Definition 2.1. (See [4].) A t-norm is a binary operation $T : [0, 1] \times [0, 1] \to [0, 1]$ that is commutative, associative, increasing in each variable, and has unit element 1.

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Definition 2.2. (See [4].) A t-conorm is a binary operation $S : [0, 1] \times [0, 1] \to [0, 1]$ that is commutative, associative, increasing in each variable, and has unit element 0.

Example 2.3. Some well-known t-norms on [0, 1] are T_M, T_P, T_L, and T_D given by, respectively:

$$T_M(x, y) = \min(x, y),$$
$$T_P(x, y) = xy,$$
$$T_L(x, y) = \max(x + y - 1, 0),$$
$$T_D(x, y) = \begin{cases} 0 & (x, y) \in [0, 1)^2, \\ \min(x, y) & \text{otherwise.} \end{cases}$$

Proposition 2.4. *(See [4].) Let I be a nonempty index set. A t-norm T is lower semicontinuous if and only if it is left-continuous in each argument, i.e., for all $x, y \in [0, 1]$ and for all $\{x_i\}_{i\in I} \subseteq [0, 1]$, $\{y_i\}_{i\in I} \subseteq [0, 1]$ we have*

$$T\left(x, \sup_{i\in I} y_i\right) = \sup_{i\in I} T(x, y_i) \text{ and } T\left(\sup_{i\in I} x_i, y\right) = \sup_{i\in I} T(x_i, y). \quad (1)$$

By the same token, the upper semicontinuity of a t-norm is equivalent to its right-continuity in each component. And then a t-norm T is continuous if and only if it is left-continuous and right-continuous.

Example 2.5. (See [4].) The following is a non-continuous t-norm:

$$T_1(x, y) = \begin{cases} \frac{xy}{2} & (x, y) \in [0, 1)^2, \\ \min(x, y) & \text{otherwise.} \end{cases}$$

Definition 2.6. (See [4].) A t-norm T is called strict if it is continuous and strictly monotone.

Definition 2.7. (See [4].) A t-norm T is called nilpotent if it is continuous and if each $a \in (0, 1)$ is a nilpotent element of T.

Definition 2.8. (See [4].) The t-norm T is called Archimedean if for each $x, y \in (0, 1)$ there exists an $n \in \mathbf{N}$ such that $x_T^{(n)} < y$.

Proposition 2.9. *(See [4].) For each Archimedean t-norm T the following are equivalent:*

i. *T is left-continuous.*
ii. *T is continuous.*

Theorem 2.10. *(See [4].) Let T be a continuous Archimedean t-norm. Then the following are equivalent:*

i. *T is nilpotent.*
ii. *There exists some nilpotent element of T.*
iii. *There exists some zero divisor of T.*
iv. *T is not strict.*

In [1], the t-norm (t-conorm) is denoted as an infix notation $\triangle$ ($\triangledown$) instead of the prefix notation $T(x, y)$ ($S(x, y)$). So, in order to be consistent with [1], $\triangle$ ($\triangledown$) denotes t-norm (t-conorm) in this note.

Definition 2.11. (See [1,4].) The residual implication $\rhd : [0, 1] \times [0, 1] \to [0, 1]$ associated with a left-continuous t-norm $\triangle$ is defined by

$$x \rhd y = \bigvee \{z | x \triangle z \leq y\}. \quad (2)$$