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Some properties and representation methods for Ordered Weighted Averaging operators

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Abstract

This study demonstrates or proposes some new properties and representation methods for Ordered Weighted Averaging (OWA) operators. The interweaving representation allows some well-known OWA operators to be decomposed into two other well-known OWA operators. A theorem is proposed for determining the relations between OWA operators with respect to their degree of orness/andness. We also provide a complex iteration method for well-known OWA operators, as well as some new generation methods and possible adjustment methods for well-known Centered OWA operators. Furthermore, we investigate some new methods for generating Step OWA and orlike/andlike window OWA operators, where these methods are accurate in terms of their orness/andness degree. We also demonstrate two interesting and useful properties of OWA operators, i.e., Stage OWA and Dew OWA, and one of their applications.

Keywords: Aggregation; Iteration; Ordered weighted averaging operator; Orness

1. Introduction

Aggregation operators play important roles in the theory of fuzzy sets. Yager [26] generalized *or* and *and* aggregation operators by introducing Ordered Weighted Averaging (OWA) operators. OWA operators have been applied in many areas such as decision making, data mining, approximate reasoning, and pattern recognition [12,17,19].

The corresponding *orness* measure, also introduced by Yager [26], plays an important role in studies of OWA operators in both theoretical and applied areas [3–6,13–21,24,26–31,36]. The orness measure reflects the orlike or andlike aggregation result of an OWA operator. There are some other important types of orness definitions and related applications [7–9], but we focus on the widely used orness/andness definitions that correspond to OWA operators.

The current trend is for OWA operators to be used only with a group of arguments, which are ordered depending on their magnitude. The corresponding result is the proposed Induced OWA operator (IOWA) [28,34,35], which allows the arguments to be reordered for aggregation, which depends on another inducing index attached to each of the arguments rather than their size. If the inducing index corresponds to a time series, it is known as a Time-induced

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OWA operator (TOWA) [11,30]. This type of operator is also used widely in recent theoretical and applied studies [11, 30].

When the new argument is derived from the original time ordered arguments vector, it generally has the newest time-inducing index due to the particularity of time series. Thus, the new argument is always added to the extreme rightmost position of the original time ordered arguments vector, which is actually equivalent to adding a smaller argument to the original ordered arguments vector under an OWA operator. In this study, we do not focus on this difference.

It is important to ensure that their orness remains unchanged as the dimensions of these TOWA operators increase gradually. In this sense, some special OWA operators are particularly appealing because they either have especially ordered and interesting structures, or because they have practical meaning in certain applications. This class of OWA operators includes the following well-known examples: Inverse Sum OWA operator [1,30], family of S-OWA operators [10,33], and the Borda–Kendall OWA (BK-OWA) operator [1,23,30]. However, if we perform aggregations directly with these OWA operators, we are generally confronted by difficult calculation especially when their dimensions increase continuously.

In this study, we propose a novel expression that allows these special OWA operators to be decomposed into two other well-known OWA operators. This method, which is called the interweaving method, illustrates some inner relations and structures of these special OWA operators, thereby providing choices for generating a new OWA operator with the given OWA operators.

When new arguments arrive sequentially on both sides of the original ordered arguments vector, the dimensions of the corresponding OWA operators increase naturally. The calculations required for the new aggregations of these OWA operators are vast if we do not use iteration methods. In general, however, there are no simple iteration forms for aggregation with those operators, thus we present a complex iteration method for the three special OWA operators mentioned above.

Furthermore, we introduce two types of weight generation methods for the Centered OWA operator [29], which is very important in theoretical and applied studies [2,15,25,32]. In general, the weights of the Centered OWA operator are located in the middle position rather than at the two ends, which is typically universal during decision making. The Centered OWA operators belong to the neutral attitudinal OWA operators, although we often have optimistic or pessimistic attitudes in practice. Therefore, to ensure that most weights are located in the appropriate attitudinal point and to take the predefined orness degree simultaneously, we provide a method for adjusting the Centered OWA to generate the desired OWA operator.

We also investigate some methods for generating Step and Window OWA operators, as well as presenting two new forms of OWA operators and demonstrating one of their applications.

The remainder of this paper is organized as follows. Section 2 provides the preliminaries regarding OWA operators. Section 3 proposes the related theorem, the interweaving expression of some OWA weighting vectors, and the new form of aggregation under those OWA operators. Section 4 mainly considers the iteration forms of these OWA operators when new arguments arrive sequentially on both sides of the original ordered arguments vectors. Section 5 reviews the Centered OWA operator and proposes two new methods for generating a Centered OWA. Next, based on the interweaving method, we propose a method for adjusting the Centered OWA to obtain the desired OWA operator. Section 6 describes two new forms of OWA operators and presents an application to education. Section 7 summarizes the main results and conclusions.

2. Preliminaries

In this section, we briefly review the OWA operators, the IOWA operators and some well-known particular cases of OWA operators.

2.1. Basic concept and properties of OWA operators and the degree of orness

The OWA operator and degree of "*orness*" were proposed by Yager in [26].

An OWA operator of dimension n is a mapping $F : \mathbb{R}^n \to \mathbb{R}$, which has an associated weighting vector $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ with the properties

$$w_1 + w_2 + \cdots + w_n = 1; \quad 0 \le w_j \le 1; \;\; j = 1, 2, \ldots, n$$