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Construction of image reduction operators using averaging aggregation functions

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Abstract

In this work we present an image reduction algorithm based on averaging aggregation functions. We axiomatically define the concepts of image reduction operator and local reduction operator. We study the construction of the latter by means of averaging functions and we propose an image reduction algorithm (image reduction operator). We analyze the properties of several averaging functions and their effect on the image reduction algorithm. Finally, we present experimental results where we apply our algorithm in two different applications, analyzing the best operators for each concrete application.

Keywords: Image reduction; Reduction operators; Local reduction operators; Aggregation functions; Averaging functions

1. Introduction

Image reduction consists in diminishing the resolution of the image while keeping as much information as possible from the original image [4,18,25]. Image reduction (and the inverse process, image magnification) is nowadays a widely studied topic due to its applicability in mobile gadgets such as phones, PDAs or cameras, where the stored images must be visualized in small screens and the need of changing the dimension of the images is very important (without significantly changing the quality of the image). Moreover, image reduction may be used as a preprocessing of the image in algorithms which are time and computationally expensive.

In the literature there exist many different image reduction methods. Some of them consider the image to be reduced globally or in a transform domain [24,27]. Some others divide the image in pieces and act on each of them locally [12,13,23]. The latter procedure allows to design algorithms which are very efficient in time and that keep some of

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the specific properties of the images such as textures, edges, etc. But in general, there is not a set of properties that the reduced image must keep from the original.

Our aim in this work is *to design a reduction algorithm that, given an image, provides a new image of lower dimension that keeps the intensity properties of the original image*. This aim has led us, firstly, to axiomatically define the concept of reduction operator as a function that takes a matrix (image) and gives back a new smaller matrix (reduced image) satisfying a minimal set of properties that, in our opinion, must be fulfilled. Moreover, we study whether some of the reduction methods in the literature are reduction operators in our sense.

To reach the main goal, we consider a second objective: *to design mechanisms to reduce small regions of an image into a single pixel that represents the intensities of the region*. In this sense, we present the so-called local reduction operators. The properties of these operators have led us to study their construction from averaging (idempotent) aggregation functions. Then, from the composition of applying the same local reduction operator to all the regions in an image, we can build a reduction operator and so achieve the main goal of the work.

In order to study the use of our reduction operators, we carry out an experimental study in two different applications. The aim is to study how the properties of several reduction operators (constructed from local reduction operators) affect the obtained reduced image, and so the result of each application. In the first application, a process of reduction–reconstruction is applied to a set of images analyzing the best reduction operator with respect to two quality measures, *PSNR* and *SSIM*. Besides, we compare the proposed reduction operators with other algorithms of the literature. In the second experiment we study the effect of reduction operators in pattern recognition problem: to estimate the face pose angle in face recognition systems.

The structure of this paper is as follows: in Section 2 we recall the main concepts used in this article. In Section 3 we present and study image reduction operators. Later, in Section 4, we define local reduction operators and we propose an image reduction algorithm based on these operators. In Section 5 we study their construction by means of aggregation functions. In Section 6 we perform an experimental study of image reduction operators. We finish with conclusions and future research in Section 7.

2. Preliminaries

We start recalling some concepts that will be used in this work.

We recall that an automorphism φ of the unit interval is any continuous and strictly increasing function $\varphi : [0, 1] \to [0, 1]$ such that $\varphi(0) = 0$ and $\varphi(1) = 1$.

Definition 1. A function $N : [0, 1] \to [0, 1]$ is a strict negation if N is strictly decreasing, continuous, $N(0) = 1$ and $N(1) = 0$. If N is also involutive (i.e., if $N(N(x)) = x$ for all $x \in [0, 1]$), then N is called a strong negation.

Theorem 1. *(See [31].) A function $N : [0, 1] \to [0, 1]$ is a strong negation if and only if there exists an automorphism φ of the unit interval such that $N(x) = \varphi^{-1}(1 - \varphi(x))$ for all $x \in [0, 1]$.*

Aggregation functions have been widely studied in recent years. An overview of them can be found in [2,9,10,16].

Definition 2. An aggregation function of dimension n (n-ary aggregation function) is a non-decreasing mapping $M : [0, 1]^n \to [0, 1]$ such that $M(0, \dots, 0) = 0$ and $M(1, \dots, 1) = 1$.

Boundary conditions and monotonicity are the minimal properties demanded to aggregation functions. However, there are other properties that can be of interest, depending on the application [5,11].

Definition 3. Let $M : [0, 1]^n \to [0, 1]$ be an n-ary aggregation function.

(i) M is said to be idempotent if $M(x, \dots, x) = x$ for any $x \in [0, 1]$.
(ii) M is said to be homogeneous if $M(\lambda x_1, \dots, \lambda x_n) = \lambda M(x_1, \dots, x_n)$ for any $\lambda \in [0, 1]$ and for any $(x_1, \dots, x_n) \in [0, 1]^n$.
(iii) M is said to be shift-invariant if $M(x_1 + r, \dots, x_n + r) = M(x_1, \dots, x_n) + r$ for all $r > 0$ and $(x_1, \dots, x_n) \in [0, 1 - r]^n$.