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Open problems from the 12th International Conference on Fuzzy Set Theory and Its Applications

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Abstract

Eighteen open problems posed during FSTA 2014 (Liptovský Ján, Slovakia) are presented. These problems concern fuzzy logics, fuzzy partitions, copulas, triangular norms and related aggregation functions. Some open problems concerning effect and MV algebras are also included.

Keywords: Copula; Effect algebra; Fuzzy implication; Fuzzy logics; Fuzzy partition; Triangular norm

1. Introduction

A public announcement of open problems had a great impact on the development of several areas of science, including mathematics. It seems so that the most famous was the formulation of D. Hilbert's problems [20]. In the domain of fuzzy sets and related topics, several open problems were published in monographs [7,31,41,48]. There are several papers devoted purely to open problems concerning triangular norms [1,30]. Other collections of open problems are linked to problems posed at conferences; recall for example the collections summarizing the open problems posed at the 2nd, 8th and 10th FSTA conferences [28,39,37]. To illustrate the influence of these collections to the development of mathematics, observe that just within the field of fuzzy sets there are more than 40 papers devoted to the solution of some of the exposed problems. The aim of this paper is the presentation of open problems posed during the conference FSTA 2014 "The Twelfth International Conference on Fuzzy Set Theory and Applications" held from January 26 to January 31, 2014 in Liptovský Ján, Slovakia.

The paper is organized as follows. In each section a brief introduction to the area of summarized open problems prepared by persons introducing these problems is given. In the 2nd section triangular norms and related negations are discussed. Section 3 is devoted to open problems on fuzzy implications. Section 4 deals with copulas. Preorders induced by uninorms are studied in Section 5. Section 6 discusses the construction of aggregation functions by means

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of penalty functions. Fuzzy partitions are discussed in Section 7, while Section 8 concerns fuzzy logics. In Section 9, several open problems on the field of universal integrals are introduced. Section 10 deals with effect algebras. States on MV-algebras are studied in Section 11. Finally some concluding remarks with e-mail addresses of authors of presented open problems are added.

2. Are Archimedean t-norms with strong associated negations left-continuous?

T-norms can be categorized under different classes based on their analytic and algebraic properties, viz., continuity, left-continuity, Archimedeanness, nilpotence, cancellativity, etc. Already some interrelationships among them are known under some conditions – for instance, it is well known that under Archimedeanness left-continuity is equivalent to continuity [33]. Similarly, when one assumes continuity of a t-norm T, many properties, which are otherwise not equivalent, become equivalent, for instance strict monotonicity of a T is equivalent to strictness, which is further equivalent to conditional cancellativity, while existence of only trivial idempotent elements becomes equivalent to Archimedeanness. For more such interrelationships please see [31].

When one considers a t-norm T with an involutive associated negation, i.e., the function

$$N_T(x) = \sup\{t \in [0,1] \mid T(x,t) \le 0\}$$

is such that $N_T \circ N_T = \mathrm{id}_{[0,1]}$, the only known result is that nilpotence is equivalent to continuity.

Our study on the mutual equivalences among the above properties under this setting [23] resulted into the following problem:

Problem 2.1 *(B. Jayaram).* Does there exist any Archimedean t-norm T, whose N_T is involutive but is not conditionally cancellative or left-continuous? In other words, is an Archimedean t-norm T whose N_T is involutive necessarily conditionally cancellative or left-continuous?

3. Lattice of fuzzy implications and the exchange principle

The exchange principle, i.e. the equation of the form

$$I(x, I(y,z)) = I(y, I(x,z)), \quad x, y, z \in [0,1], \tag{EP}$$

where $I: [0,1]^2 \to [0,1]$, generalizes the classical tautology

$$p \to (q \to r) \equiv q \to (p \to r)$$

and is one of the most important properties of a fuzzy implication both from theoretical and applicational point of view (see [2]). Unfortunately, in general, (EP) is not preserved by standard lattice operations minimum and maximum. For a counterexample see [2, Remark 6.1.5], where it is shown that the Goguen implication

$$I_{\mathbf{GG}}(x,y) = \begin{cases} 1, & \text{if } x \le y, \\ \frac{y}{x}, & \text{if } y > x, \end{cases}$$

and the Reichenbach implication

$$I_{\mathbf{RC}}(x,y) = 1 - x + xy$$

satisfy (EP), but fuzzy implications $I_{\mathbf{GG}} \vee I_{\mathbf{RC}}$ and $I_{\mathbf{GG}} \wedge I_{\mathbf{RC}}$ do not satisfy (EP). In particular, this implies that if I, J are two (S,N)-implications, then $I \vee J$ and $I \wedge J$ are not necessarily (S,N)-implications. One can easily check that the same holds for R-implications generated from left-continuous t-norms, or f- and g-implications.

Problem 3.1 *(M. Baczyński, B. Jayaram).* Characterize the subfamily of all fuzzy implications ((S,N)-implications, R-implications, etc.) which preserve the (EP) for lattice operations.