Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Sets and Systems 274 (2015) 4–11
FUZZY sets and systems
www.elsevier.com/locate/fss

Fuzzy optimization: Milestones and perspectives

M.K. Luhandjula

Department of Decision Sciences, University of South Africa, PO Box 392, UNISA, Pretoria 0003, South Africa

Available online 24 January 2014

Abstract

We provide a condensed and selective look at the known landscape of the theory and application of Fuzzy optimization, emphasizing Professors' Tanaka and Asai contribution. Significant ideas are picked out in the trajectory of this field and projections are made for its future developments. From our discussions, it unquestionably emerges that the above mentioned Japanese academics have played a noticeable role in shaping the subject from its genesis to its maturation. Moreover, their research work will continue to inspire prospective works in this field.

Keywords: Optimization; Mathematical programming; Uncertainty; Fuzzy sets

1. Introduction

Optimization is a very old and classical area which is of high concern to many disciplines. Engineering as well as Management, Politics as well as Medicine, Artificial Intelligence as well as Operations Research and many other fields are in one way or another concerned with optimization of designs, decisions, structures or information processes. In addition to this, many concrete problems that may be cast into an optimization setting are rife with sources of imprecision. Without any claim for exhaustivity, we may mention: uncertainty related to errors, sparsity of data, subjectivity of expert judgements and ambiguity inherent to the range of parameter values. In this connection, the noted philosopher Nietzche was quoted as saying: "No one is gifted with immaculate perception." Moreover, false certainty is bad science and it could be dangerous if it stunts articulation of critical choices.

It is well known that, in order to tackle a difficult problem, an algorithm has to incorporate all the available information. Neglecting some inherent feature like imprecision would, according to the Computer Science rule "garbage in, garbage out", leave no other chance to the algorithm but to churn out meaningless outcomes.

The foregoing clearly indicates the persistent need to improve optimization models realism by making it possible to incorporate uncertainty into mathematical programming frameworks. This has given rise to the realm of Mathematical programming under uncertainty.

Mathematical disciplines like Stochastic programming, Robust optimization, Grey programming, Interval programming, hybrid Rough set–Particle swarm optimization have arisen from the above mentioned need.

E-mail address: luhanmk@unisa.ac.za.

0165-0114/$ – see front matter
http://dx.doi.org/10.1016/j.fss.2014.01.004

This paper considers a more recent approach to Optimization under uncertainty in which imprecision is modeled by fuzzy relations and/or fuzzy parameters. It briefly surveys part of the known landscape of the theory and application of Fuzzy optimization.

In such a short span of time and space, it is virtually an impossible task to do justice to the whole field of Fuzzy optimization. We have then, deliberately, chosen to bring across some glimpses with emphasis to Professors' Tanaka and Asai contribution.

We take a more discursive approach, reflecting more on ideas rather than on technical details.

For a detailed picture of Fuzzy optimization, we refer the reader to several textbooks [1–3] and to references therein. An interested reader may also consult authoritative overviews by Inuiguchi, Ichihashi and Tanaka [4], Rommelfanger [5], Inuiguchi and Ramik [6] and Delgado et al. [7].

2. Flexible programming

In solving their problems, Decision makers generally grapple with technological, environmental and competitive factors which interact in a complicated fashion. In such a turbulent environment, the formulation of the problem in terms of dichotomous "yes or no" statements yields often inconsistencies which are expressed by the vacuousness of the feasible set.

More than this, a Decision maker may feel more comfortable formulating his optimization problem using expressions of the "more or less" type.

The foregoing gives rise to the need for models that accept leeways on the achievement of the goal and the constraints in an optimization setting.

Fuzzy set theory is of particular importance in the formulation and solution of optimization problems, with relaxed goals and constraints. As a matter of fact, such flexible goals and constraints may be properly modeled using the language of this theory.

Making use of Bellman–Zadeh's confluence principle [8], one may single out a satisfying solution in this context. It is worth mentioning that in such a turbulent environment, the "optimum optimorum" no longer exists hence the term "satisfying solution" suits better than "optimal solution".

The above outlined approach has a long and active history dating back to the seminal paper by Tanaka, Okuda and Asai [9] in mid-1970's. In that paper a mathematical program with a fuzzy objective function and fuzzy constraints was clearly formulated. Authors then exploited, with good reason, properties of α-level sets of involved fuzzy sets to handle, theoretical, algorithmic and practical issues related to the fuzzy mathematical problem under scrutiny.

This line of research was further developed by the same authors [10,11] and by Zimmermann [12], giving birth to the subfield of Fuzzy Optimization called Flexible programming.

These research works coupled with advances in Computing Technology sparked a massive flurry of interest in Flexible programming in the 1980's. This decade together with the next one saw also successful applications of Flexible programming across a wide spectrum of domains including: water resource management, air-pollution reduction and media selection.

It is worth mentioning that the main idea behind Flexible programming is to consider all goals and constraints as flexible constraints. The use of the minimum operator, instead of the sum, enforcing every constraint or goal to be somewhat respected. As a consequence, one may consider Flexible Optimization as pioneering soft constraint satisfaction in Artificial Intelligence (see e.g. [13,14]).

This idea of symmetry between the goal and the constraints of a mathematical program, that is used in an essential manner in Flexible programming, is not to everybody's taste [15]. Some researchers are against this symmetric approach. They accept the idea of representing soft goals and constraints by appropriate fuzzy sets but they stop short of taking the additional leap of considering the goal and the constraints as identical concepts. They advocate that constraints should delineate the feasible set while the goal should remain on its role of ranking feasible alternatives.

3. Fuzzy robust programming

Another Fuzzy optimization model that came to the fore during the 1970's is the fuzzy robust one [16]. Instead of specifying the feasibility of an action by a set of inequalities, like in conventional optimization, in Fuzzy robust