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ScienceDirect

Fuzzy Sets and Systems 272 (2015) 70–88

FUZZY sets and systems

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A practical approach for estimating weights of interacting criteria from profile sets

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Received 11 November 2013; received in revised form 1 October 2014; accepted 12 January 2015

Abstract

Aggregating multi-criteria data is an important problem with many applications. Commonly-used additive aggregation methods, such as the weighted arithmetic mean, cannot account for the criteria interactions encountered in many practical multi-criteria decision problems. The Choquet integral is a suitable aggregation operator in the presence of interacting criteria. It replaces the weight vector with a fuzzy measure that models the importance of each subset or coalition of criteria, rather than just the importance of individual criteria. However, estimating the fuzzy measures in practice has been problematic. Conventional approaches are cognitively challenging for decision makers, while more recent approaches suffer from prohibitive data requirements. In this paper, we present a formulation for the weights of the Choquet integral that uses principal component analysis to account for criteria interaction. This novel unsupervised approach to estimating the required fuzzy measures overcomes the limitations of other methods. The approach is applied to two case studies in environmental and sustainability analysis, and the results are compared with those of the weighted arithmetic mean. The first case is a triple bottom line analysis of 135 Australian industry sectors evaluated against 11 criteria, while the second case is an environmental life cycle assessment of 8 alternative biosolids management options evaluated against 5 criteria. These examples demonstrate the ability of the proposed approach to account for criteria interaction in these and other decision contexts.

Keywords: Multiple criteria evaluation; Non-additive measures; Decision analysis; Multivariate statistics; Environment; Sustainability; Interacting criteria; Choquet integral

1. Introduction

The choice of a set of criteria against which alternatives are evaluated is itself a value judgment in all multi-criteria decision methods, not only with regard to which criteria are selected, but also with regard to how the criteria are defined, and in how much detail they are represented. In particular, in practical applications of multi-criteria de-

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http://dx.doi.org/10.1016/j.fss.2015.01.011
0165-0114/

cision methods, variables cannot generally be constructed without interaction (i.e. dependant relationships among criteria). For example, in a multi-criteria environmental systems analysis, the criteria ‘eutrophication potential’ and ‘NOx emissions’ convey some common information because eutrophication by NOx is represented in both criteria. However, each criterion also conveys independent information: eutrophication by other substances, and the other environmental impacts of NOx (e.g. human health impacts), respectively. Careful selection of criteria can often improve this situation, but some interaction will generally remain.

Commonly-employed aggregation procedures such as the arithmetic mean and simple additive weighting are unable to account for criteria interaction. The results of decision analyses with interacting criteria may therefore be contended, because issues represented by more than one criterion are likely to have a disproportionately high influence on the final outcome [1,2]. If, for example, a triple bottom line (economic, environmental and social sustainability) analysis considers more economic criteria than environmental criteria, then those alternatives with strong economic performance will be given an automatic advantage under a conventional aggregation scheme.

During the past three decades, a method has been developed that provides a way of aggregating interacting criteria. The Choquet integral [3] is a fuzzy integral which is based on a non-additive measure and can be used as an aggregation operator when criteria interact. However, determining weights in practice has been an often revisited topic. The conventional approaches are cognitively challenging for decision makers, while more recent approaches suffer from a so-called ‘curse of dimensionality’ in terms of their data requirements (Section 2.3). The aim of this paper is to provide a novel solution to these problems. It presents a formulation for the weights of the Choquet integral that uses principal component analysis (PCA) to account for criteria interaction.

The following section briefly reviews the mathematics of fuzzy integrals,[1] and describes existing approaches to estimating the weights of the Choquet integral. After this, the equivalent number of non-interacting criteria is introduced, and an intuitive interpretation is given with the help of simple examples. The new measure is then applied to two case studies. Finally, conclusions are drawn about the applicability and limitations of the proposed method.

2. Background

2.1. The Choquet integral

Consider a finite set of decision alternatives $X = \{a_1, \dots, a_m\}$ and a finite set of criteria $C = \{c_1, \dots, c_n\}$ against which the alternatives are evaluated.

Each alternative $a_k \in X$ is associated with a *profile* of *partial scores* $p^k = (p_1^k, \dots, p_n^k) \in \mathrm{R}^n$ where, for all $i = 1, \dots, n$, p_i^k is the evaluation of alternative a_k with respect to criterion c_i, with $p_i^k \in P_i \subseteq \mathbb{R}$ [4,5].

Assume further that the evaluations are defined according to the same interval scale $(P_i = P \forall i)$ [5].[2] For example, it is common to scale the evaluations to the range [0, 1] by a linear transformation. This assumption implies that all the attributes are commensurate [8].

Within many multi-criteria decision methods that allow criteria interaction and interdependence, the problem is to compute a *global score* $M(p^k)$ for each alternative $a_k \in X$, which takes into account all of the partial scores $p^k = (p_1^k, \dots, p_n^k)$ and the *weights* representing the importance of the criteria. An aggregation operator $M : \mathbb{R}^n \to \mathbb{R}$ is used for this purpose [8–10].

One of the most commonly used aggregation operators is the weighted arithmetic mean [4,8,9]. This operator gives the global score $M(p)$ associated with the profile $p = (p_1, \dots, p_n)$ as

$$M_\omega(p) = \sum_{i=1}^{n} \omega_i p_i \tag{1}$$

where, for all $i = 1, \dots, n$, ω_i is the *weight* of criterion c_i with $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$ [8].

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[1] The following review is non-exhaustive and deals only with the aspects relevant to this problem.

[2] The scale transformation mentioned here is not trivial. However, this topic is not within the scope of this research article. The interested reader may refer to [6] or [7].